Preliminary Examination: Electricity and Magnetism

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Instructions:

- This exam consists of 10 problems with 10 points each.
- Read all 10 problems before you begin to solve any problem, and solve the problems that seem easiest to you first. Spend your time wisely. If you are stuck on one problem, move on to the next one, and come back to it if you have time after you have solved all other problems.
- Show necessary intermediate steps in each solution. Partial credit will be given if merited.
- No textbook, personal notes or external help may be used other than what is provided by the proctor.
- This exam takes 3 hours.

Potentially Useful Information:

Physical constants and symbols:

ϵ	permittivity,	ϵ_0	vacuum permittivity,
μ	permeability,	μ_0	vacuum permeability,
e	electric charge of the proton,	m_e	mass of the electron,
$c = 1/\sqrt{\epsilon_0\mu_0}$	speed of light,		
ρ	electric charge volume density,	$\mathbf{j}$	electric current density.

Formulas and relations:

- Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \qquad \nabla \times \mathbf{H} = \mathbf{j}_{\text{free}} + \partial_t \mathbf{D},$$

  where $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E}$ and $\mathbf{H} = \mu_0^{-1}\mathbf{B} - \mathbf{M} = \mu^{-1}\mathbf{B}$.

- Bio-Savart's law for the magnetic field at position $\mathbf{r}$ due to a steady current element $I\mathrm{d}\boldsymbol{\ell}'$ located at position $\mathbf{r}'$:

$$\mathrm{d}\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{I\mathrm{d}\boldsymbol{\ell}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}.$$

- $Z\mathbf{H} = \hat{\mathbf{k}} \times \mathbf{E}$ and $k = n\omega/c$ for a monochromatic plane wave in medium, where $Z = \sqrt{\mu/\epsilon}$ is the impedance, and $n = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$ is the refractive index.
- The radiation magnetic field at position $\mathbf{r}$ due to a time-varying electric dipole $\mathbf{p}$ at the origin:

$$\mathbf{B}(t, \mathbf{r}) = -\frac{\mu_0}{4\pi rc}\hat{\mathbf{r}} \times \ddot{\mathbf{p}}(t - r/c).$$

Problem 1: A uniformly charged ring of radius R and charge Q is centered at the origin and rotates in the x-y plane about the z axis with angular velocity ω. Determine the electric and magnetic fields at the center of the ring.

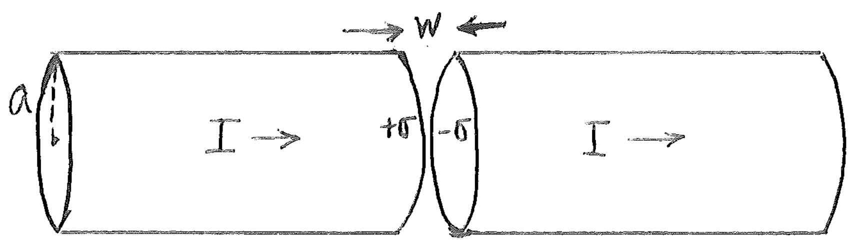
Problem 2: A solid cylinder of radius R and height h is polarized along its axis (chosen to be the z axis) with polarization density $\mathbf{P}$ increasing linearly with height from a value $P_1\hat{\mathbf{z}}$ at the bottom face to $P_2\hat{\mathbf{z}}$ at the top face. Calculate the bound surface charge density and the volume charge density. Show by integrating the total bound charge densities that there is no net charge in the cylinder.

Problem 3: A monochromatic electromagnetic wave travels in a material with permittivity ϵ_1 and permeability μ_1. It comes to an interface with a second material at normal incidence with permittivity ϵ_2 and permeability μ_2. Use the boundary conditions at the interface derived from Maxwell's equations to calculate the ratio between the amplitudes of the electric fields in the two materials. What fraction of the power of the incident wave is transmitted?

Problem 4: A point charge q is situated at the center of a dielectric spherical shell of permittivity ϵ_1 with inner and outer radii a and b, respectively. The space at $r > b$ is vacuum, and the space at $r < a$ is filled with another dielectric material of permittivity ϵ_2. Determine the electric field $\mathbf{E}$ in all regions.

Problem 5: A point charge q moves with a constant velocity $\mathbf{v}$ in a space filled with constant and uniform electric field $\mathbf{E} = E_0\hat{\mathbf{x}}$ and magnetic field $\mathbf{B} = B_0\hat{\mathbf{y}}$. Find or constrain the three components of the velocity of the point charge (v_x, v_y, v_z).

Problem 6: A capacitor is formed by a small gap w in a wire of radius a where $w \ll a$. The initial surface charge density σ on either side of the gap is 0. Starting at $t = 0$, a constant current I flows in the wire. Find the electric and magnetic fields in the gap. Ignore fringing fields.



Problem 7: Derive the charge density that produces the electrostatic potential of the form

$$V(x,y,z) = V_0 \left\{ \exp\left[\left(\frac{x}{x_0} \right)^2 \right] \cos\left(\frac{y}{y_0} \right) + \left(\frac{z}{z_0} \right)^2 \right\},$$

where (x, y, z) are Cartesian coordinates, and x_0, y_0 and z_0 are three constants.

Problem 8: A parallel plate capacitor consists of two plates of area A in the vacuum separated by a variable distance d. The capacitor is connected to a battery with voltage V. Calculate the force of one plate on the other as a function of V and d. Further show that the external work that must be performed to slowly increase the separation distance from $d = d_1$ to $d = d_2$ is given by the negative change in $CV^2/2$, where C is the capacitance.

Problem 9: Find the magnetic field $\mathbf{B}(r, \theta, z)$ that is generated by the following current density $\mathbf{j} = j_0 e^{-r^2/r_0^2} \hat{\mathbf{z}}$, where j_0 and r_0 are constants, and (r, θ, z) denote the cylindrical coordinates.

Problem 10: A point charge q moves along a circular orbit of radius R with angular speed ω. Another point charge $-q$ also moves along the same orbit with the same speed as q but is on the opposite side of the circle. Determine the instantaneous rate of energy loss to the lowest order. What is the condition under which your result is a good approximation of the total radiation power?