Department of Physics and Astronomy, University of New Mexico

Electricity and Magnetism Preliminary Examination

Fall 2015

Instructions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- NO cheat sheets are allowed.
- Total time: 3 hours.

Useful Formulas and Relations

- Maxwell's equations:

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0}\,,$$
$$\vec{\nabla}.\vec{B} = 0\,,$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\,,$$
$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)\,.$$

- Biot-Savart law for the magnetic field at position $\vec{r}$ due to a steady current element $Id\vec{l}'$ at position $\vec{r}'$:

$$\vec{B}(r') = \frac{\mu_0}{4\pi}\frac{Id\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,.$$

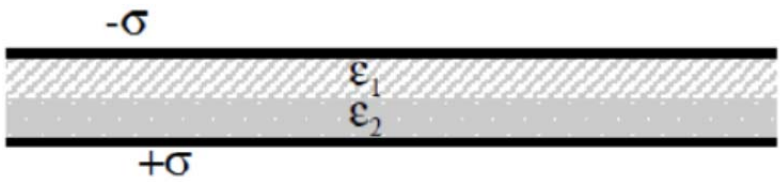
- Time-averaged power radiated by an oscillating electric dipole:

$$P = \frac{\mu_0|p|^2\omega^4}{12\pi c}\,.$$

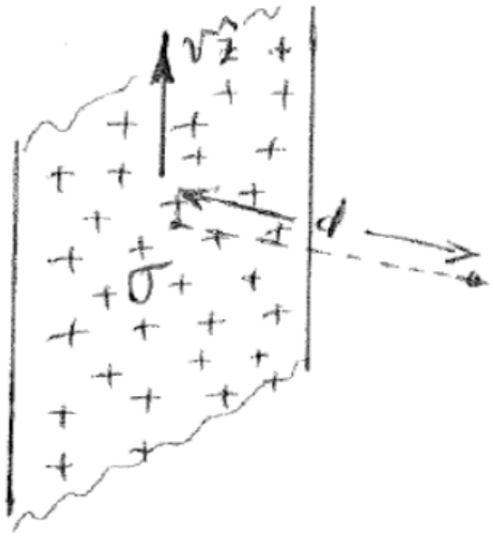
1– In the Cartesian coordinates (x, y, z), the electrostatic potential has the form $V = a|y|$, where a is a constant. Derive the specific charge density that produces this potential.

2- The positive terminal of a battery (ground taken at infinity) is attached to an initially neutral perfectly conducting sphere of radius R as shown below. Find the total charge on the sphere when equilibrium is established. How much work does the battery do in this process?

3– A parallel plate capacitor is filled with two equal thickness layers of dielectrics of permittivity $\epsilon_1 = 2\epsilon_0$ and $\epsilon_2 = 1.5\epsilon_0$ respectively. A surface charge density $\pm\sigma$ is placed on the two plates. Find the electric field inside each of the dielectrics, and the bound surface charge density at the surface of the plates and at the interface between the two dielectrics.



4– An infinitely extended plane with a uniform charge density σ has a uniform velocity v in the z direction as shown below. Find the magnitude and direction of the magnetic field at a distance d on either side of the plane.

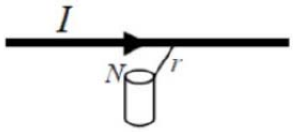


5– An electron with speed $v << c$ undergoes cyclotron motion in the xy plane in a magnetic field $B(r)$ along the z direction. The speed of the electron is increased without changing the cyclotron radius r_0 by slowly varying the B field in time. Show that this is possible if

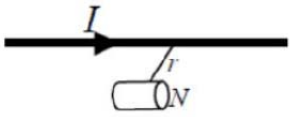
$$B(r_0, t) = \frac{\int_0^{r_0} B(r,t) r dr}{r_0^2}.$$

What causes the increase in the kinetic energy of the electron?

6– A bar magnet with magnetic dipole **m** is placed at a radial distance r from an infinitely long wire carrying a current I. Find the magnitude and direction of the force exerted on the magnetic dipole for the two orientations shown below. The distance r is much greater than the length of the magnet.



(i) **m** perpendicular to I and r.

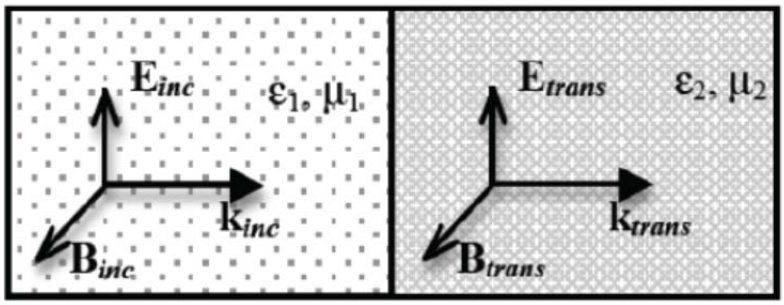


(ii) **m** parallel to I, perpendicular to r.

7– A monochromatic electromagnetic wave travels with permittivity ϵ_1 and permeability μ_1. It comes to an interface with a second material at normal incidence with permittivity ϵ_2 and permeability μ_2. Use the boundary conditions the interface and show that the ratio of the transmitted to the incident electric field amplitude is

$$\frac{E_{trans}}{E_{inc}} = \frac{2Z_2}{Z_1 + Z_2},$$

where $Z_i = \sqrt{\mu_i/\epsilon_i}$ is the wave impedance in the material. What is the fraction of the transmitted power (i.e., transmittance)?



8– A monochromatic electromagnetic wave $\vec{E}(\vec{r}, t)$ travels through a neutral plasma and induces a current density $\vec{J} = -n_e e\vec{v}$, where n_e is the density of the electrons driven with instantaneous velocity $\vec{v}$ by the electric field. Use Maxwell's equations to show that the electric field inside the plasma obeys the following wave equation:

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{E}(\vec{r}, t) = \frac{\omega_p^2}{c^2}\vec{E}(\vec{r}, t),$$

where $\omega_p = \sqrt{n_e e^2/m_e\epsilon_0}$ is the plasma frequency (m_e is the electron mass).

9- Two circularly polarized plane waves of opposite helicity but with the same frequency, wave vector, and amplitude are coherently superposed. Show that the resulting plane wave is linearly polarized and find the direction of polarization in terms of $\Delta\phi \equiv \phi_+ - \phi_-$ (ϕ_+ and ϕ_- are the phase constants of the two circularly polarized waves). What would be the polarization of the resulting plane wave if the circularly polarized wave had the same helicity?

10– Two point charges $\pm q$ undergo uniform circular motion of radius R and angular frequency ω. Under what condition involving R and ω will the charges emit predominantly electric dipole radiation? Find the instantaneous rate of energy loss due to electric dipole radiation. Why does the charge pair not emit any magnetic dipole radiation at all?