Department of Physics and Astronomy, University of New Mexico

E&M Preliminary Examination

Fall 2011

Instructions:

- The exam consists of 10 problems (10 pts each).
- Partial credit will be given if merited.
- Personal notes on two sides of an 8.5"x 11" sheet are allowed.
- Total time: 3 hours.

Possibly Useful Formulas

- Relation of spherical polar coordinates, (r, θ, ϕ), to Cartesian coordinates:

$$x = r \sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi, \quad z = r \cos\theta.$$

  Unit vectors:

$$\hat{r} = \sin\theta \cos\phi \, \hat{x} + \sin\theta \sin\phi \, \hat{y} + \cos\theta \, \hat{z};$$

$$\hat{\phi} = -\sin\phi \, \hat{x} + \cos\phi \, \hat{y}; \quad \hat{\theta} = \hat{\phi} \times \hat{r}.$$

- Electric field at position $\vec{r}$ due to a point electric dipole of moment $\vec{p}$ located at the origin:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\frac{3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}}{r^3} \right].$$

- Magnetic field at position $\vec{r}$ due to a point magnetic dipole of moment $\vec{m}$ located at the origin:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \left[\frac{3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}}{r^3} \right].$$

- Biot-Savart Law for the magnetic field at position $\vec{r}$ due to a steady current element $I d\vec{\ell}'$ located at position $\vec{r}'$:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}.$$

- Time-averaged power radiated by an oscillating electric dipole:

$$P = \frac{\mu_0 |p|^2 \omega^4}{12\pi c}.$$

- Time-averaged power radiated by an oscillating magnetic dipole:

$$P = \frac{\mu_0 |m|^2 \omega^4}{12\pi c^3}.$$

- Time-averaged power radiated by a non-relativistically moving charge with acceleration a (Larmor formula):

$$P = \frac{q^2 a^2}{4\pi \epsilon_0 c^3}.$$

- Fresnel formulas for the amplitude reflection coefficient of a plane wave incident at a planar interface between two dielectrics:

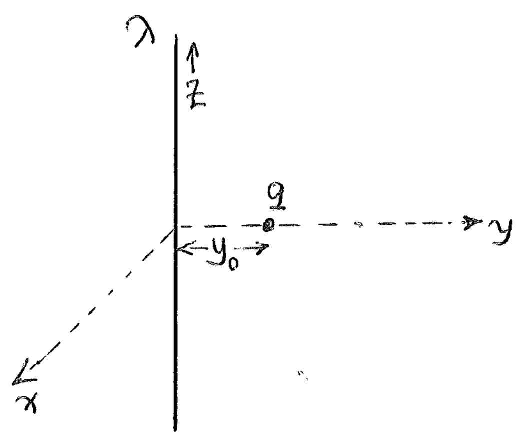
$$r_\perp = \frac{n \cos\theta - n' \cos\theta'}{n \cos\theta + n' \cos\theta'}; \quad r_\parallel = \frac{n' \cos\theta - n \cos\theta'}{n' \cos\theta + n \cos\theta'},$$

where $\perp, \parallel$ refer, respectively, to polarizations perpendicular and parallel to the plane of incidence. The angles of incidence and refraction are θ and θ', and n, n' are the refractive indices of the medium of incidence and the medium of transmission, respectively.

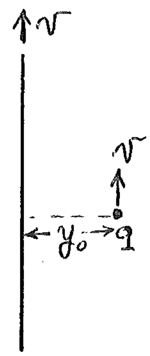
1. A thin infinitely extended conducting slab carries a surface charge density σ on both its plane surfaces. What is the electric field (direction and magnitude) everywhere in space? If a second identical plate of the same initial charge density is brought from infinity to a short distance d from the first plate and is oriented parallel to the first plate, then what is the electric field everywhere now? How is the charge density on either plate modified due to the presence of the other plate?

2. A point charge q of mass m is placed a distance y_0 from an infinitely massive, uniformly charged, infinitely extended straight wire of charge density λ (of the same sign as q) per unit length. Derive by means of the work energy theorem the following integral relation between the distance y of the charge from the wire and the time t after the charge is released from rest:

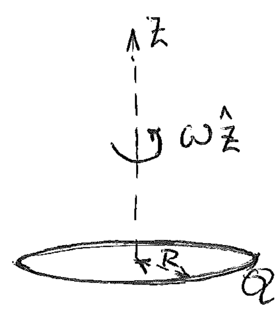
$$\int_1^{y/y_0} \frac{du}{\sqrt{\ln u}} = \sqrt{\frac{q\lambda}{\pi\epsilon_0 m}}\, y_0 t.$$



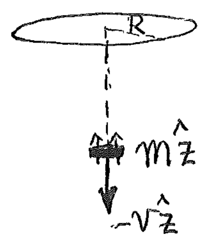
3. Describe in detail how the motion of the point charge q will change in the previous problem if both the charge and the uniformly charged wire were to be set into motion with a uniform speed v along the length of the wire at $t = 0$. Derive, in particular, the position and velocity of the charge at time t in terms of the given parameters and the usual electromagnetic constants. Treat all motion fully relativistically. (*Hint:* It may be useful to analyze the problem in a frame that is moving with the wire and then Lorentz-transform back to the laboratory frame.)



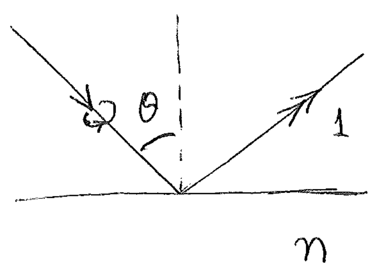
4. A uniformly charged ring of radius R, charge Q, and centered at the origin in the xy plane is rotated at angular velocity $\omega\hat{z}$ about its axis. What are the electric and magnetic fields on the axis of the ring a distance z away from its plane? Show that at large distances, $z >> R$, the electric and magnetic fields may be approximated by those due to their lowest-order multipoles.



5. A circular ring of radius R and electrical resistance Z is located in the xy plane and is centered at the origin. A tiny disk magnet of magnetic dipole moment $m\hat{z}$ is located a distance L below the ring on the z axis, as shown. What is the magnetic flux due to the magnet threading the ring? If the magnet were to move at velocity $-v\hat{z}$ away from the ring, then what would be the current induced in the ring at that instant? (*Hint:* The substitution $u = (s^2 + L^2)$ may be useful to transform an integral you may encounter to a simple calculable form.)



6. A circularly polarized plane electromagnetic wave of angular frequency ω is incident from air at angle θ to the surface normal on a plane transparent slab of refractive index n. For what angle of incidence, as a function of n, will the reflected wave be fully linearly polarized? At other angles of incidence, what is the state of polarization of the reflected light?



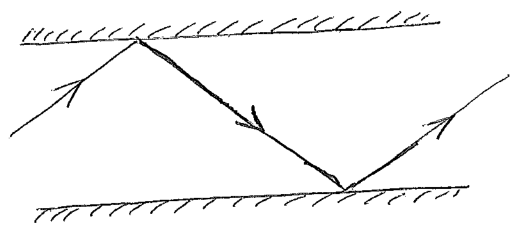
7. A plasma interacting with a static magnetic field $\vec{B}_0$ along the direction of propagation of an electromagnetic wave of frequency ω presents two slightly different indices of refraction to the two opposite circular polarizations of the wave,

$$n_{\pm} = \sqrt{1 - \frac{\omega_P^2}{\omega^2 \mp \omega\omega_B}}, \quad \omega_B \equiv \frac{eB_0}{m}, \tag{1}$$

where $-e$ and m are the electron charge and mass, respectively, and ω_P is the plasma frequency. What is the group velocity of the wave if it is (i) left-circularly polarized (upper sign in the above expression) or (ii) right-circularly polarized (lower sign)? If the initial wave is a wave packet that is linearly polarized, then show that two wave packets, each of a pure circular polarization, result after sufficiently long propagation through the plasma. For how long a time will the initial wave packet need to travel before the peaks of the two resulting circularly polarized wave packets have separated by a distance l? Will the resulting wave packets have the same length as the initial wave packet? Why or why not?

8. An alternating current $I(t) = I_0 \cos(\omega t)$ flows in a small square loop of side a. How small does a have to be so the radiation from the square loop may be well approximated as being magnetic dipole radiation? If the resistance of the loop is R, then calculate the ratio of the time-averaged power dissipated into Joule heat and that radiated by the loop.

9. Electromagnetic radiation of frequency ω is confined to propagate in a thin glass slab of refractive index n and thickness d. The glass plate is silvered to a highly metallic finish on both its faces. What is the minimum value of ω for which confined propagation can occur by means of repeated reflections of an obliquely propagating plane wave by the two slab faces?



10. An electron (charge: $-e$, mass: m) passes through a thin gold foil. During its passage in the forward direction, its speed decreases from v to 0 due to an electromagnetic braking force of form $-\alpha v$, where α is a positive constant. Calculate the total time-averaged energy radiated by the electron during its motion, assuming that $v << c$. Neglect gravity.