Department of Physics and Astronomy, University of New Mexico

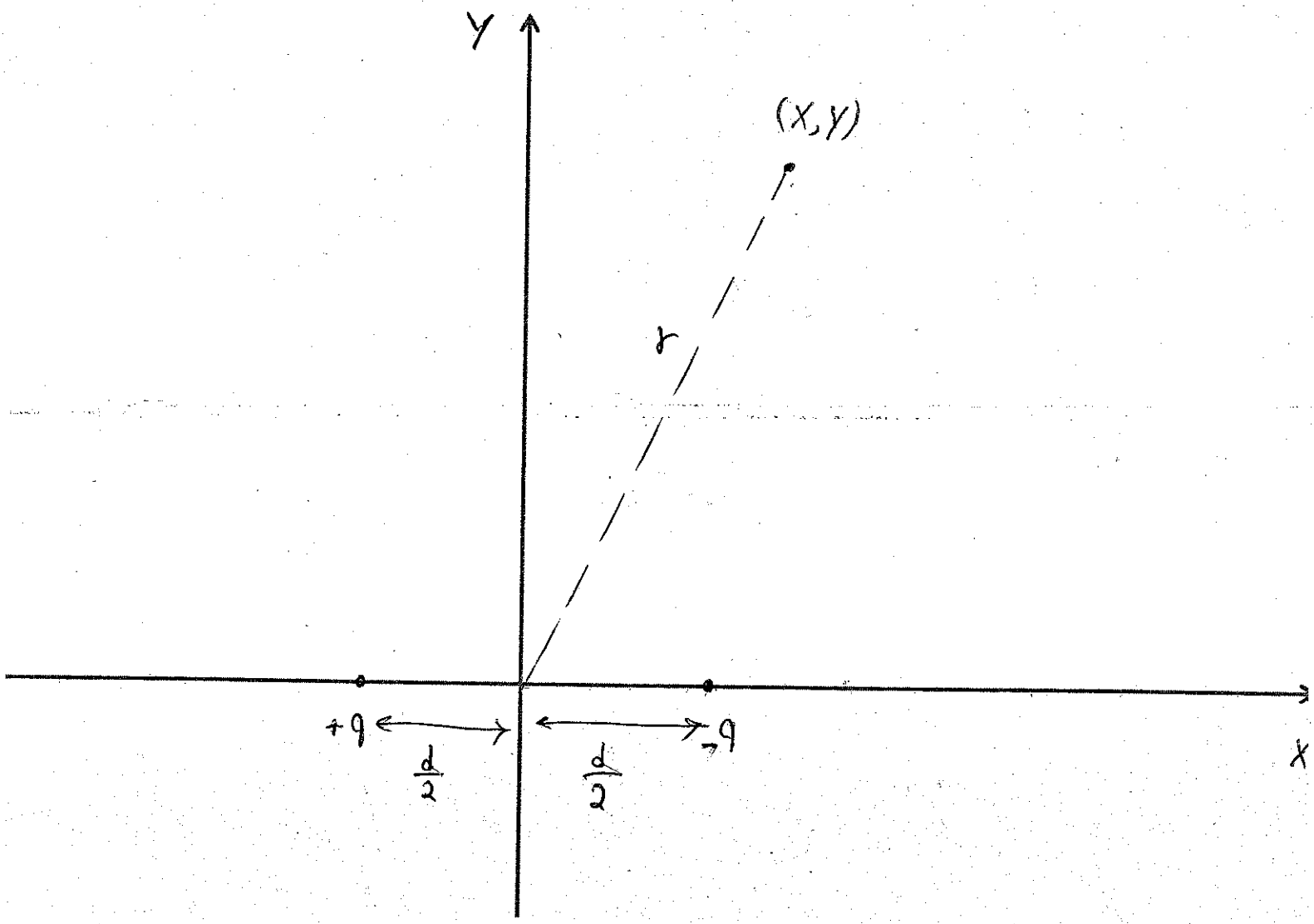
Electricity and Magnetism Preliminary Examination

Fall 2009

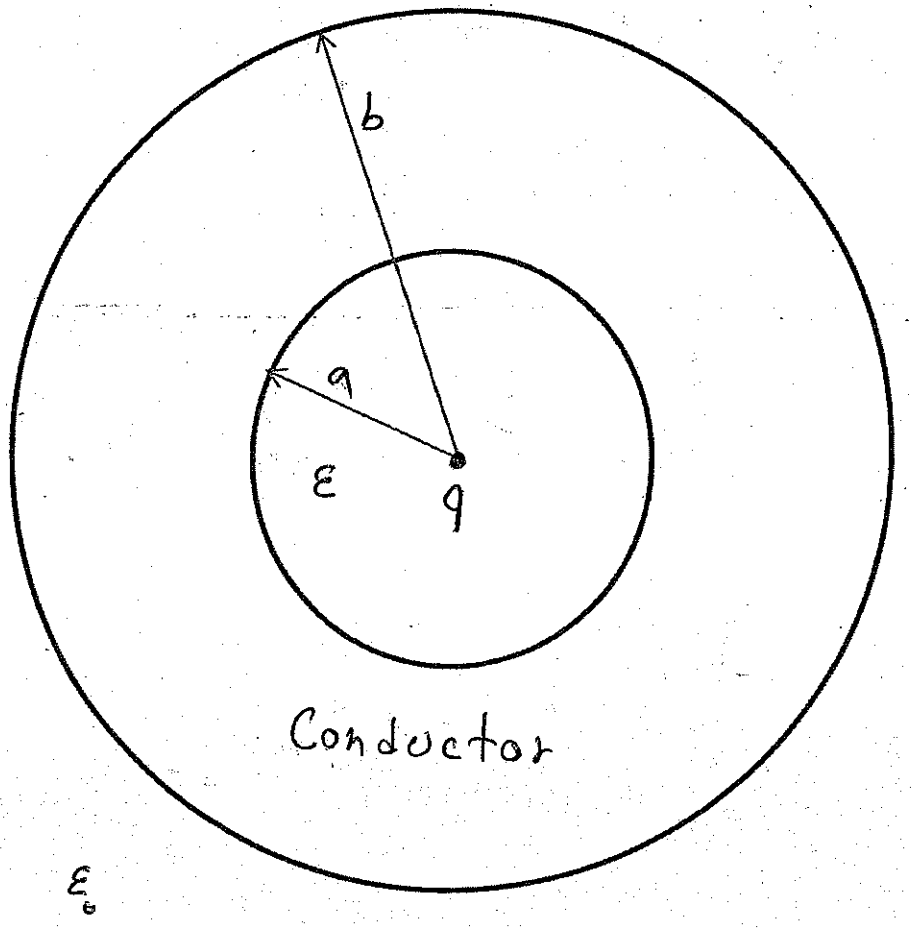
Instructions:

- The exam consists of 10 short-answer problems (10 points each).
- Partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed.
- Total time: 3 hours.

1– Two charges $+q$, $-q$ separated by a distance d are situated in the xy plane as shown in the figure. Find the potential $\Phi(x, y)$ at any point in the xy plane. What is the leading order contribution to Φ when $r \gg d$? (r is the distance of the point (x, y) from the origin)

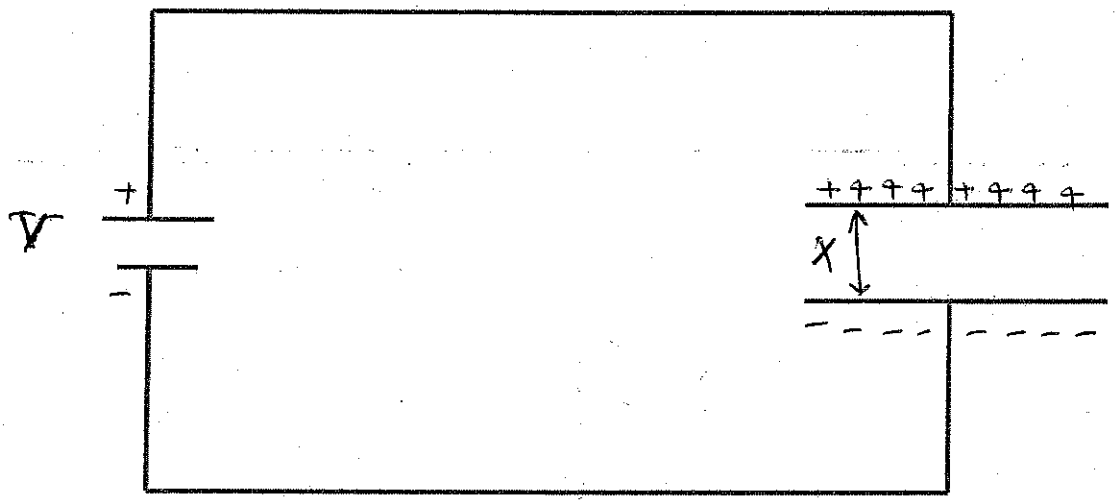


2– A point charge q is situated at the center of a conducting spherical shell of inner and outer radii a and b. The space at $r < a$ is filled with dielectric material with permittivity ϵ. Determine the potential Φ in all regions.

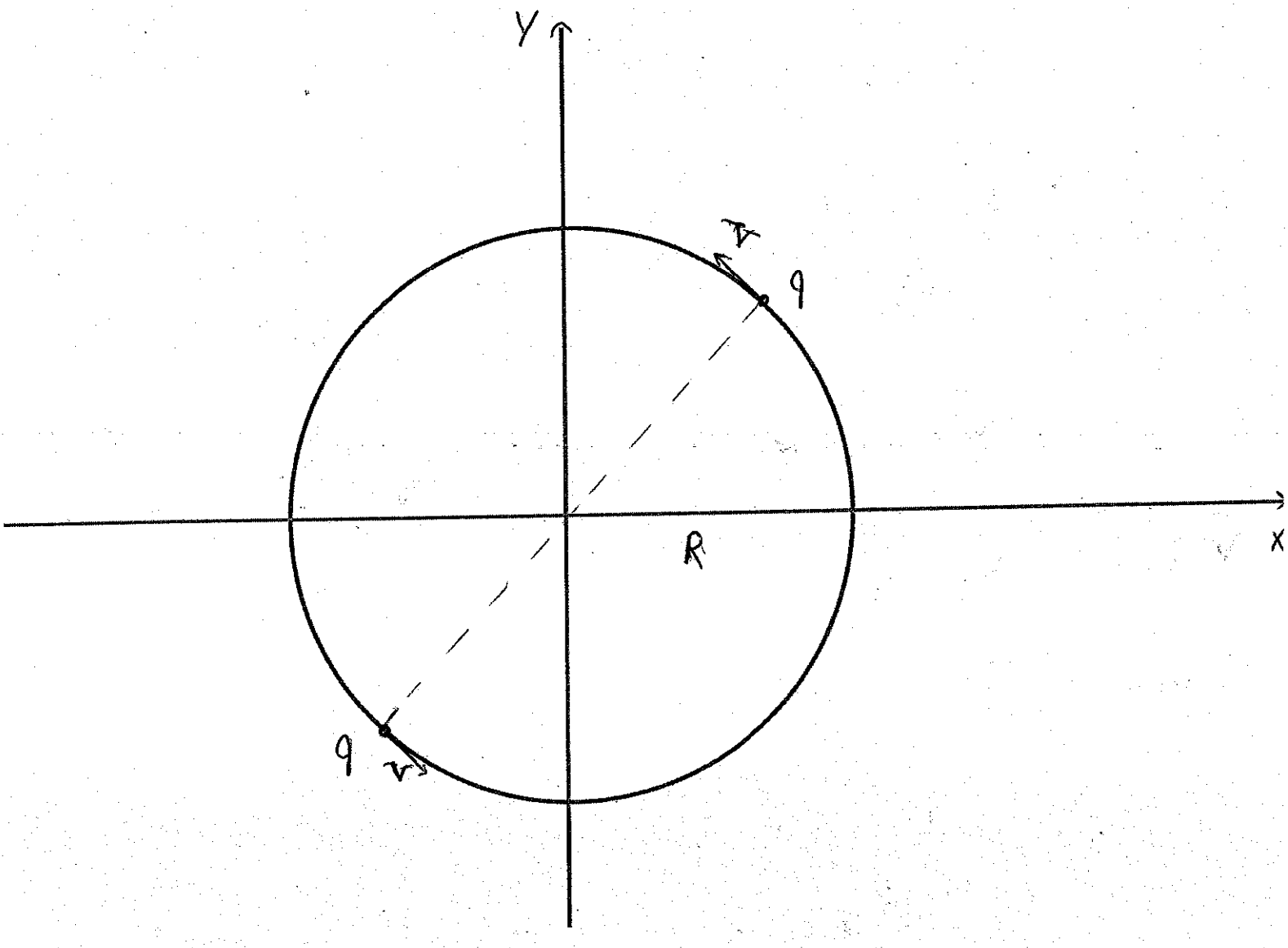


3– A capacitor consisting of two parallel plates of unit area is connected to a battery of voltage V. The distance between the two plates is x. The electrostatic energy stored in the capacitor is $U = Q^2/2C$, where electric charge on the two plates are $\pm Q$.

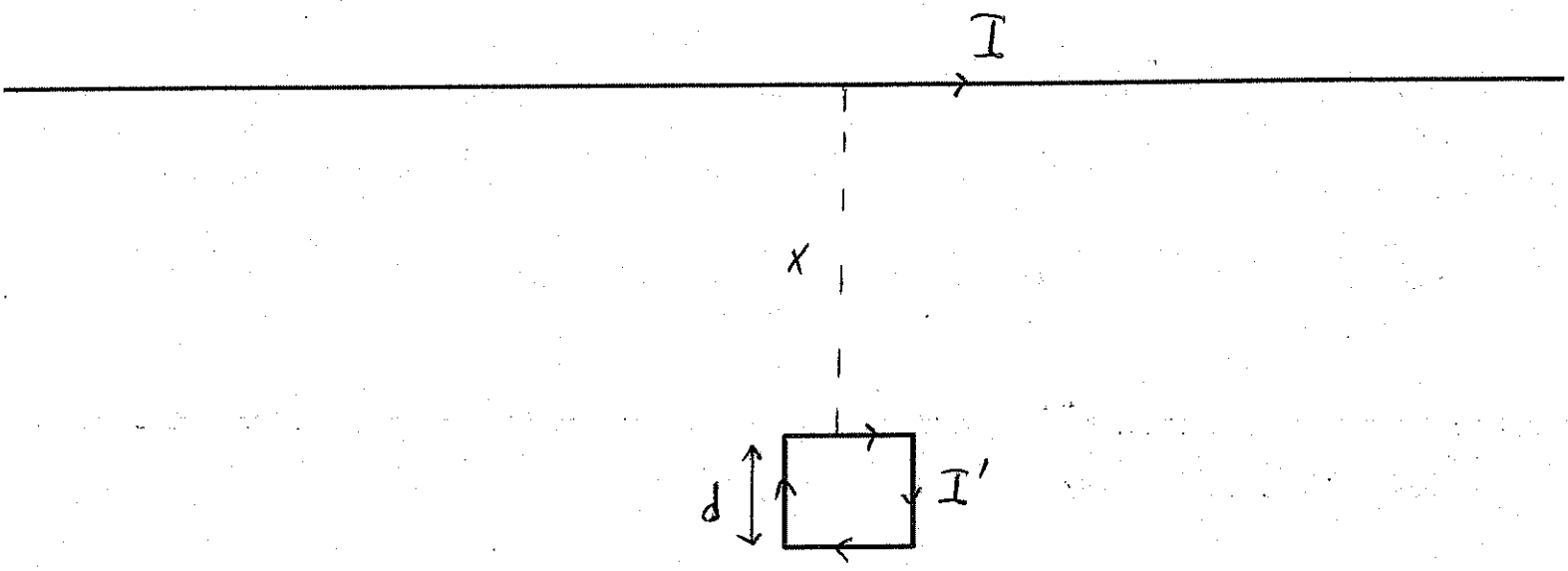
Show that the energy stored in the capacitor decreases as x increases (capacitor remains connected to the battery at all times). How do you reconcile this with the fact that the plates of capacitor attract each other?



4– Two point charges q are rotating on a circle of radius R around the origin in the xy plane. Find the magnetic induction $\vec{B}$ at a point $(0, 0, z)$ on the z axis.

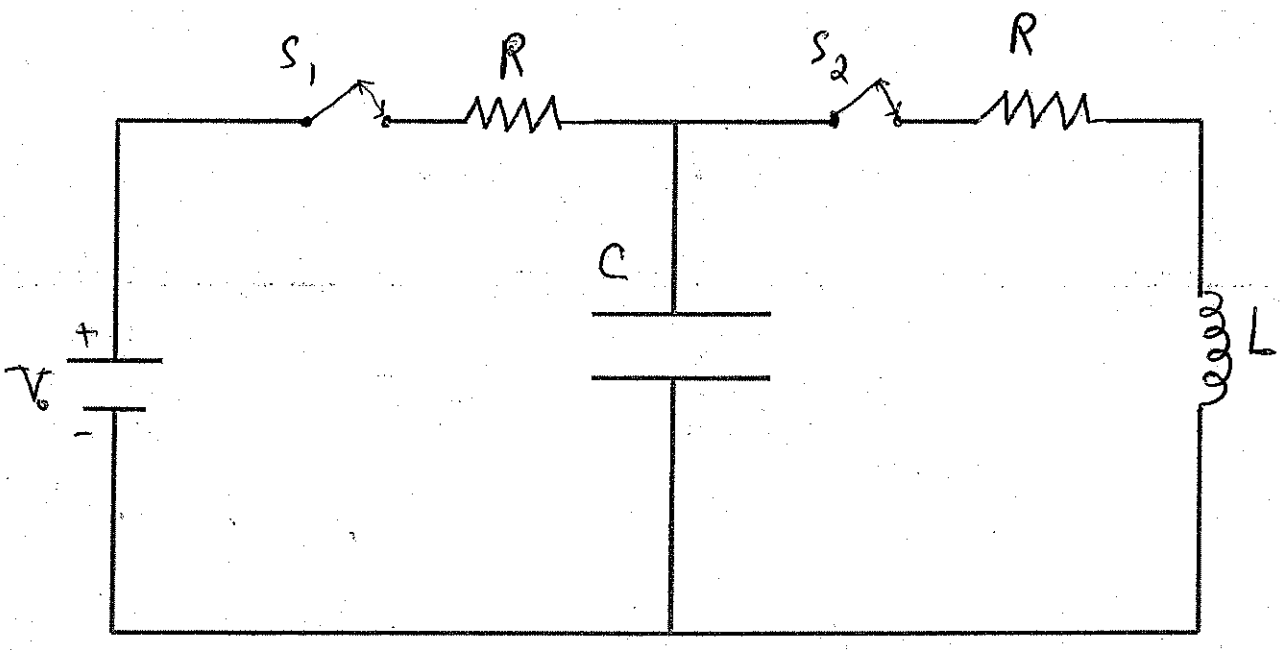


5– Consider a square loop of wire with side length d that is oriented parallel to a long straight wire carrying current I. The loop carries current I' as shown in the figure. Determine the force exerted on the loop. (Both currents are held constant)



6– Consider the circuit in the following figure. Both switches s_1, s_2 are open initially. Switch s_1 is closed at $t = 0$, while s_2 is still open. At a time $T \gg \sqrt{RC}$ switch s_1 is opened and s_2 is closed.

Sketch a graph of the potential of the capacitor as a function of time (assume that $R^2 \ll 4L/c$). Calculate the total energy dissipated as heat from $t = 0$ to ∞.



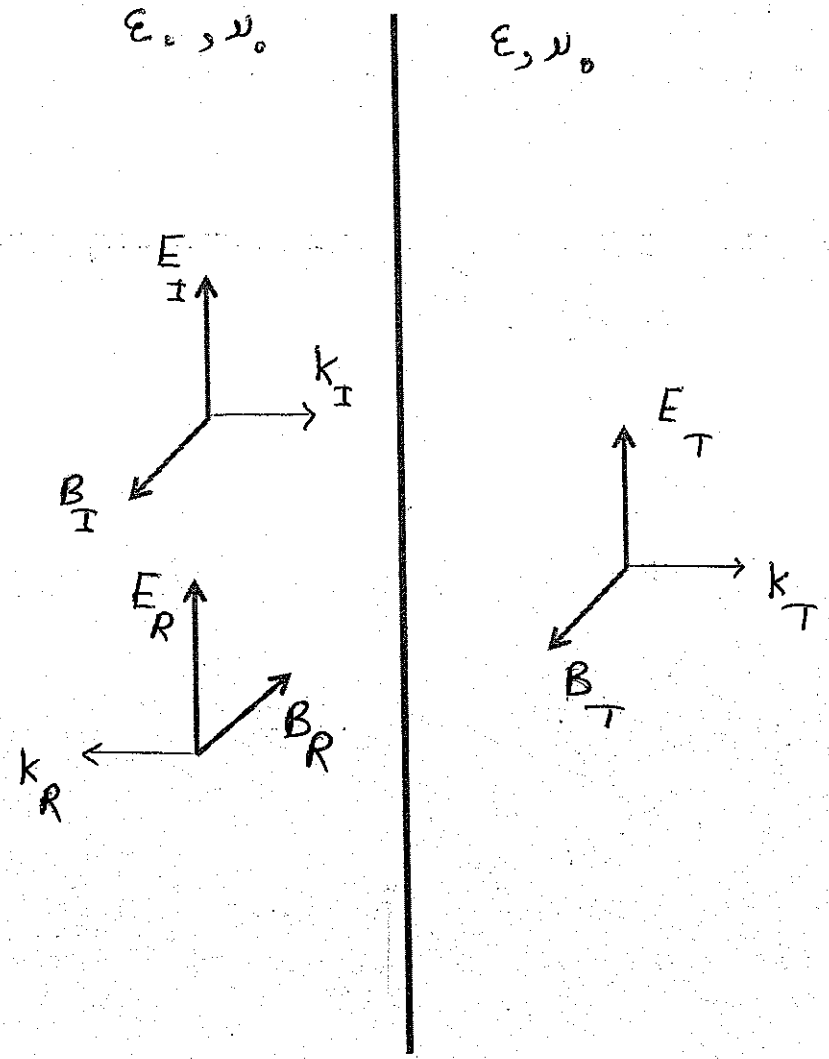
7– An electron with speed v undergoes cyclotron motion (in the xy plane) in a magnetic field $B(r)$ (along the z direction) at the cyclotron radius $r_0 = mv/eB(r_0)$. The speed of the electron can be increased by slowly increasing the B-field in time. Show that if the field at r_0 is half of the average across the orbit

$$B(r_0, t) = \frac{1}{2} \frac{\int B(r,t) da}{\pi r_0^2}$$

the radius of orbit is constant in time. (Assume non-relativistic speeds)

8– Consider a plane electromagnetic wave of amplitude E_I that is normally incident on a non-magnetic, non-conducting dielectric with permittivity ϵ.

Use the boundary conditions on $\vec{E}$ and $\vec{B}$ fields at the interface to find the amplitude of the reflected and transmitted waves (denoted by E_R and E_T respectively). Find ϵ such that the two amplitudes are equal.



9– Two plane waves of frequency ω are moving through vacuum and the corresponding electric fields are given by

$$\vec{E}_1 = \hat{y}E_0\cos(kz - \omega t) \quad , \quad \vec{E}_2 = \hat{y}E_0\cos(kz + \omega t)\,.$$

Find the total electric field $\vec{E}_3$, the total magnetic field $\vec{B}_3$, and the time-averaged Poynting vector for the entire system.

10– An electromagnetic wave travels inside a neutral plasma where the density of free electrons is n_e. It can be shown from Maxwell's equations that this wave satisfies the equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{\omega_p^2}{c^2}\vec{E},$$

where $\omega_p = (n_e e^2/\epsilon_0 m)^{1/2}$ is the plasma frequency.

Find and sketch the dispersion relation (equation relating the wave number k to the frequency ω) for a plane wave of frequency ω inside the plasma. Describe the propagation of the wave for $\omega < \omega_p$ and $\omega > \omega_p$.