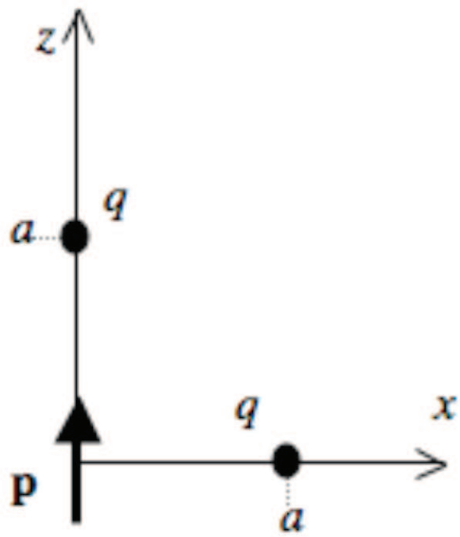
Preliminary Examination: Electricity and Magnetism
Department of Physics and Astronomy
University of New Mexico
Fall, 2007

Instructions:

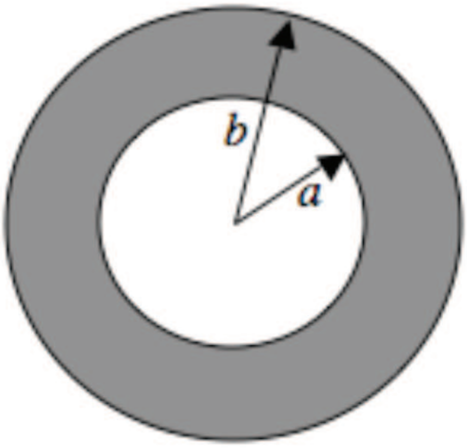
- The exam consists of 10 problems, 10 points each.
- Partial credit will be given if merited.
- Personal notes on two sides of an 8×11 page are allowed.
- Total time is 3 hours.

1. An ideal electric dipole of moment $\vec{p} = p\hat{z}$ is situated at the origin. What is the force, caused by the dipole, on each of two separate point charges, of amount q. The first is located at a distance a from the origin along the $\hat{x}$-axis, i.e., so that the charge has the Cartesian coordinates $(a, 0, 0)$, and the other is also at a distance a from the origin, but along the $\hat{z}$-axis, i.e. so that the charge has the Cartesian coordinates $(0, 0, a)$?



2. Please find the capacitance per unit length of two coaxial, hollow, metal, cylindrical tubes, of radius a and $b > a$.

3. A hollow sphere carries charge density $\rho = c/r^2$ in the region $a \le r \le b$. Find the electric field in each of the three regions: within the hollow of the sphere, i.e., for $r \le a$; within the interior of the sphere, i.e., for $a \le r \le b$; and exterior to the sphere, i.e., for $b \le r$. Provide the result in terms of the total charge, q, of the shell. Provide a plot of the magnitude of the electric field as a function of the distance r from the center of the system.



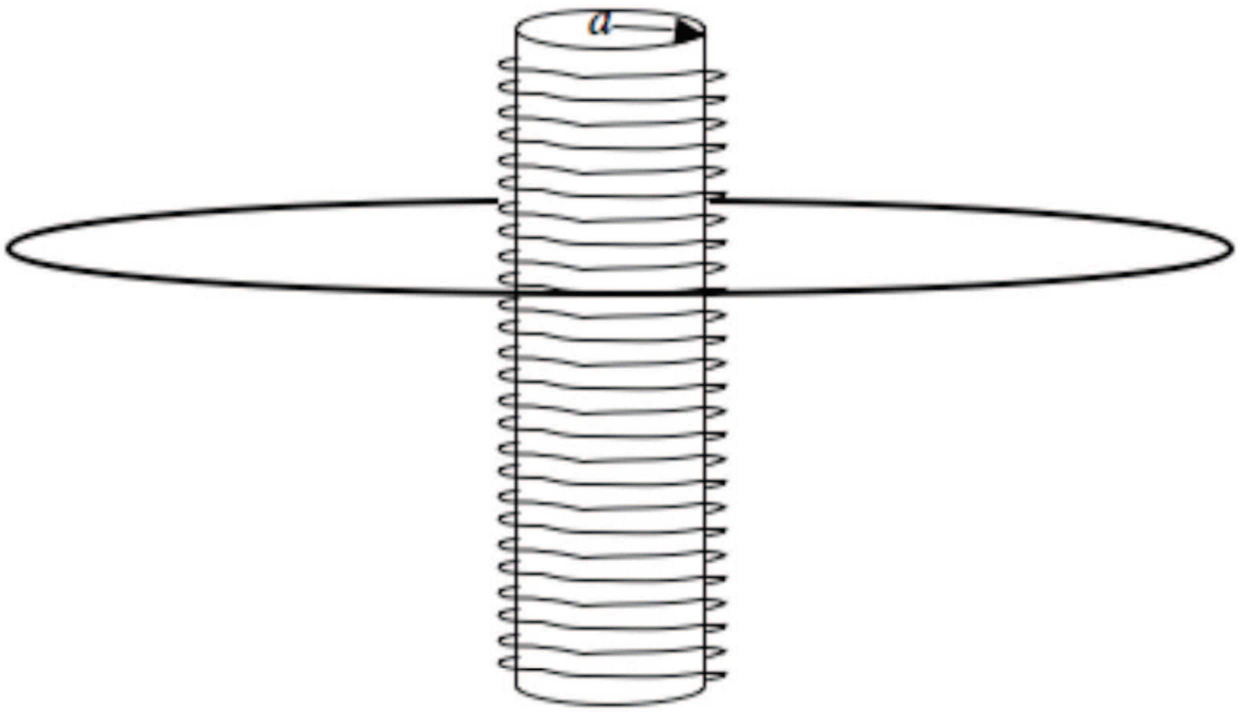
4. A uniformly charged shell of surface charge density σ and radius a is rotating at a constant angular velocity $\vec{\omega}$, and we take the $\hat{z}$-axis along $\vec{\omega}$. At an arbitrary location, $\vec{r}$, it has a magnetic vector potential given by

$$\vec{A}(r,\theta,\varphi) = \begin{cases} \frac{1}{3}\mu_0 R\sigma\omega\, r\sin\theta\,\hat{\varphi}\ ,\ r \le R\ , & \text{i.e., inside the shell,} \\ \frac{1}{3}\mu_0 R^4\sigma\omega\,\dfrac{\sin\theta}{r^2}\hat{\varphi}\ ,\ r \ge R\ , & \text{i.e, outside the shell.} \end{cases}$$

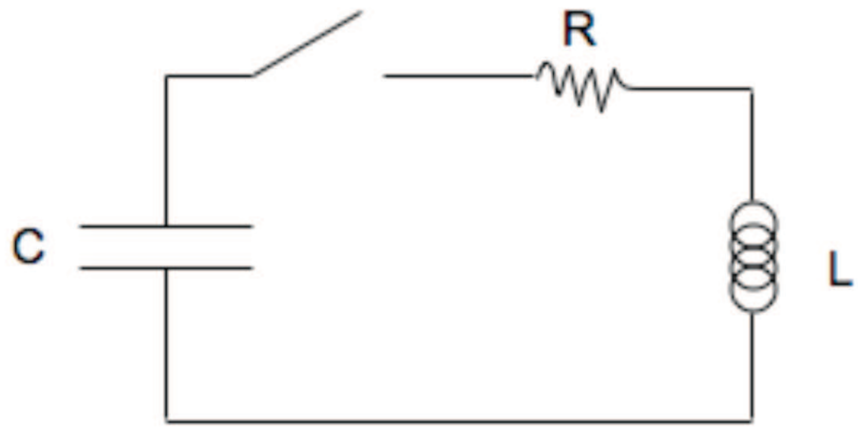
Show that the magnetic field inside the rotating shell is uniform, and along the $\hat{z}$-direction. Also determine the magnetic field outside the shell. Can you describe that field in simple language? Is the field continuous at the boundary of the shell? Explain physically your answer.
Note that for a vector of the form $\vec{A} = A\,\hat{\varphi}$, one has the following relation for its curl:

$$\nabla \times (A\,\hat{\varphi}) = \frac{\hat{r}}{r\sin\theta}\frac{\partial(A\sin\theta)}{\partial\theta} - \frac{\hat{\theta}}{r}\frac{\partial(Ar)}{\partial r}\ .$$

5. A very long solenoid carries a current I. Coaxial with the solenoid is a large, circular ring of wire, with resistance R. When the current in the solenoid is gradually decreased, a current is induced in the ring. Take the solenoid to have n turns per unit length, and radius a, while the ring has radius $b >> a$. What is the current in the ring, as a function of dI/dt?



6. A previously-charged capacitor, of amount C and charge separation Q, is in a simple open circuit along with a resistor, R, and an inductor, L. At time $t = 0$, a switch is closed so that this circuit now constitutes a single, closed, series circuit. What is the time dependence of the current through the resistor?



7. Consider a monochromatic wave moving through vacuum, of frequency ω, and with an electric field that is the sum of two separate parts, which are presented here in their complex forms:

$$\widetilde{\vec{E}} = \widetilde{\vec{E}}_1 + \widetilde{\vec{E}}_2 \ ,$$

$$\widetilde{\vec{E}}_1 = E_0 \hat{z}\, e^{i(kx-\omega t)} \ , \qquad \widetilde{\vec{E}}_2 = -E_0 \hat{z}\, e^{-i(kx+\omega t)} \ ,$$

where E_0 is real.
Determine the associated, real-valued magnetic field, and the time-averaged Poynting vector for the entire wave system. Please explain the meaning of your result for the Poynting vector.

8. Consider a circularly-polarized electromagnetic plane wave, propagating in vacuum with frequency ω. Write down the complex form for the electric field, and then, before you perform any averages over cycles, determine the (real-valued) intensity for the wave, making comments about the time dependence of the result.

9. At a certain time, which we take to be $t = 0$, we turn on the current, everywhere at once, in a particular infinitely-long wire, so that the current in this wire may be expressed in the following way:

$$I(t) = \begin{cases} 0 \ , & t < 0 \ , \\ I_0 \ , & t \geq 0 \ . \end{cases}$$

(We take the wire to lie along the $\hat{z}$-axis.)
At any later time, $t > 0$, and at any particular measurement location, say a distance s directly away from the wire, only some finite portion of the wire can have communicated to this observation point the information that there is now a current running in that portion of the wire. For such a given positive time, t, and distance s, what total length of wire can have communicated this information?

10. An electromagnetic plane wave of frequency ω is traveling in the $\hat{x}$-direction through the vacuum. It has amplitude E_0, is polarized in the $\hat{y}$-direction, and has (time-averaged) intensity I_0. The observer, $\mathcal{S}$, who made these statements is at rest. However, she sees another observer, $\mathcal{S}'$, coming past her, moving in the same direction as the plane wave, at half the speed of light.
What are the frequency and intensity of the wave as seen by this other observer, $\mathcal{S}'$?