Department of Physics and Astronomy, University of New Mexico

Classical Mechanics Preliminary Examination

Spring 2018

Instructions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- NO cheat sheets are allowed.
- Total time: 3 hours.

Useful Constants, Formulas, and Relations:

- Mass of the Earth: $M_E = 5.97 \times 10^{24}$ kg, Radius of the Earth: $R_E = 6.37 \times 10^3$ km
- Gravitational constant: $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
- Moment of inertial of a uniform solid sphere of mass M and radius R: $I = \frac{2}{5}MR^2$
- Moment of inertial of a uniform rod of mass M and length l about its center of mass: $I = \frac{1}{12}Ml^2$
- Moment of inertia of a uniform rectangular plate of mass M, length l, and width w situated in the xy plane about its principal axes:

$$\begin{pmatrix} \frac{1}{12}Ml^2 & 0 & 0 \\ 0 & \frac{1}{12}Mw^2 & 0 \\ 0 & 0 & \frac{1}{12}M(l^2+w^2) \end{pmatrix}$$

- Euler's equation for a rotating rigid body:

$$\frac{d\vec{L}}{dt} = \frac{\partial \vec{L}}{\partial t} + \vec{\omega} \times \vec{L}$$

- The relation between the energy E and momentum p of an object with mass m ($p \equiv |\vec{p}|$):

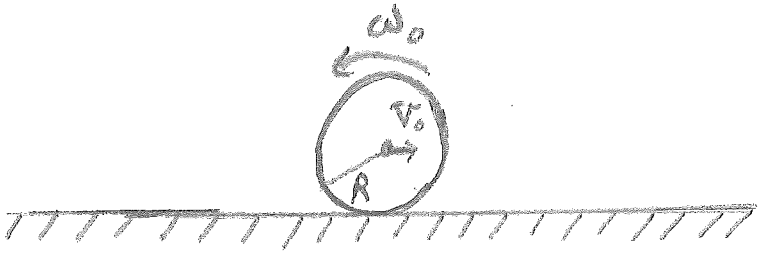
$$E^2 = (pc)^2 + (mc^2)^2$$

1– A geostationary orbit is a circular orbit around the Earth where the orbital period of an object is equal to the Earth's rotational period. Find the radius of such an orbit in kilometers.

2- A uniform rod of length l and mass m stands vertically on a frictionless floor and then tips over. Find the velocity of the rods center of mass when it hits the floor.

3– A rocket (starting from rest) moves in free space and ejects exhaust with constant relative veolcity u. Find the velocity of the rocket when its mass is reduce to half if its initrial value m_0.

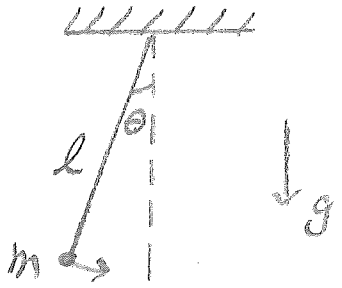
4– A sphere of radius R with uniform density hits the floor with an initial linear velocity v_0 and initial angular velocity ω_0 as shown in the figure. The coefficient of kinetic friction between the sphere and the floor is μ_k. What is the final linear velotiy of the ball?



5- A simple pendulum is made of a rigid, massless rod of length l and a point mass m as shown in the figure. The pendulum moves at small angle and experiences a damping force

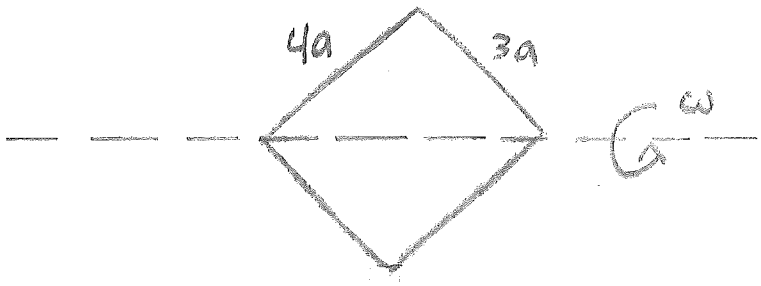
$$\vec{F} = -\beta m(gl)^{1/2}\dot{\theta}\hat{\theta}\,,$$

where β is a positive, dimensionless constant. Determine the values of β for which the pendulum is overdamped.

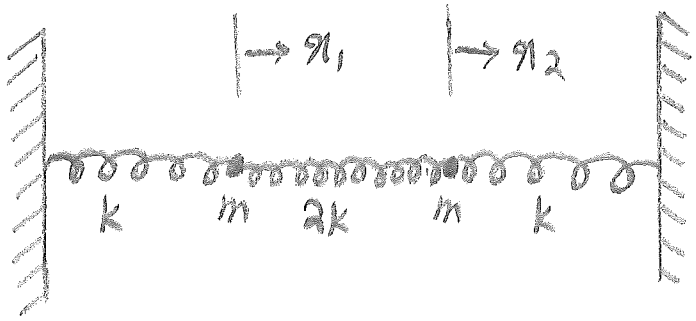


6- Consider an attractive central force $F(r) = -k/r^5$. Find the radius of the circular orbit of an object with mass m angular velocity l, and show that the orbit is always unstable.

7– A uniform rigid rectangle of mass M, length $4a$, and width $3a$ is rotated about one of its diameters as shown in the figure. Find the torque that must be supplied in order for the angular velocity to be constant at the instant shown.



8- Two identical masses m are connected to each other and to two walls by three springs as shown in the figure. Using the displacements x_1 and x_2 of the masses from their equilibrium positions, write the equations of motion for the two masses. Find the normal modes and frequencies of this system.



9– A bead of mass m moves without friction along a stiff wire bent in the shape if a helix $z = \alpha R\phi$ under the influence of gravity. Write a Lagrangian in termns of the azimuthal angle ϕ and find the equation of motion for $\phi(t)$.

10– A neutral pion π_0 decay to two photons. Suppose that π_0 moves with speed v in the $+x$ direction, and that the photons are emitted in the $+x$ direction and $-x$ direction respectively. Find v such that $E_+ = 2E_-$.