Department of Physics and Astronomy, University of New Mexico

Classical Mechanics Preliminary Examination

Spring 2017

Instructions:

- This exam consists of 10 problems with 10 points each.
- Read all 10 problems before you begin to solve any problem, and solve the problems that seem easiest to you first. Spend your time wisely. If you are stuck on one problem, move on to the next one, and come back to it if you have time after you have solved all other problems.
- Show necessary intermediate steps in each solution. Partial credit will be given if merited.
- No textbook, personal notes or external help may be used other than what is provided by the proctor.
- This exam takes 3 hours.

Useful Constants, Formulas, and Relations

- Moment of inertial of a uniform solid sphere of mass M and radius R: $I = \frac{2}{5}MR^2$
- Moment of inertial of a uniform rod of mass M and length l about its center of mass: $I = \frac{1}{12}Ml^2$
- Effective potential in the radial direction for a central potential $V(r)$ (l is the angular momentum):

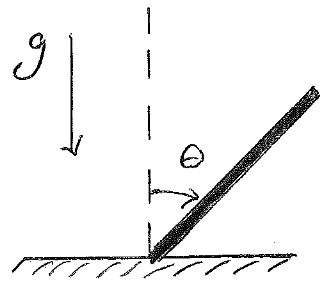
$$V_{eff}(r) = V(r) + \frac{l^2}{2mr^2}.$$

- Euler's equation for a rotating rigid body:

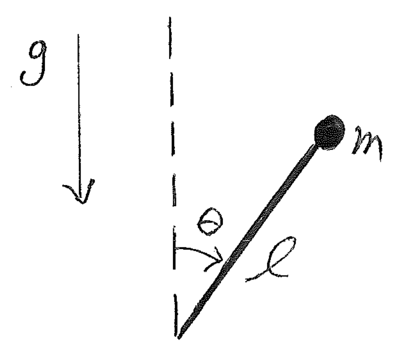
$$\frac{d\vec{L}}{dt} = \frac{\partial \vec{L}}{\partial t} + \vec{\omega} \times \vec{L}$$

1– A thin pencil with length 20 cm is balanced on a desk top, standing on its point so that its angle of inclination θ with respect to the vertical is nearly zero. Suppose that initially $\theta(0) = 1 \times 10^{-16}$ and $\dot{\theta}(0) = 0$. Assuming that the pencil point is fixed, how long does it take for the pencil to tip over on its side (i.e., $\theta = 90°$)?
Hint: You may find the following integral, which is valid for $\theta_0 < 0.01$, useful:

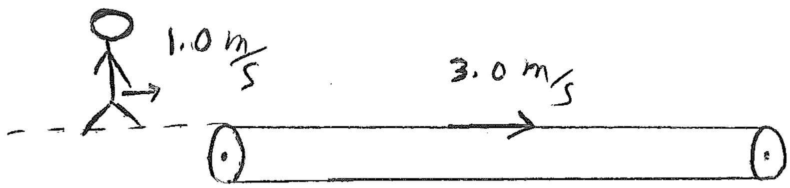
$$\int_{\theta_0}^{\pi/2} \frac{d\theta}{\sqrt{\cos\theta_0 - \cos\theta}} \simeq -\sqrt{2}\ln\theta_0 + 1.695\,.$$



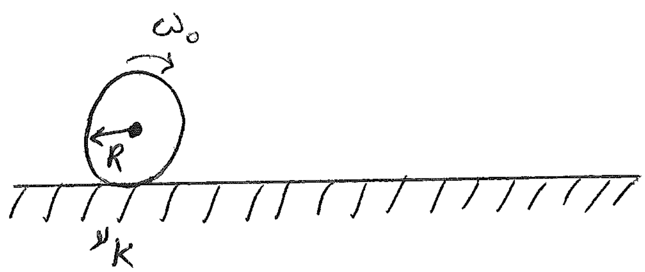
2- A particle of mass m is attached to a massless rigid rod of length l and rotates in a vertical plane with constant angular speed ω as shown in the figure. Find the magnitude of the force that is applied to the mass by the rod as a function of angle θ.



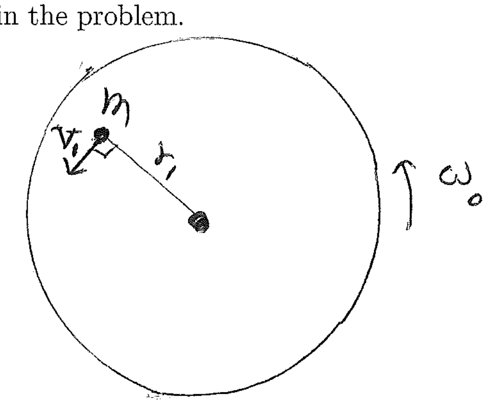
3– A moving walkway has a constant velocity of 3.0 m/s. Pedestrians step on to the walkway at a speed of 1.0 m/s relative to the ground. If an average pedestrian has a mass of 65 kg, and 20 passengers step on the walkway every minute, find the average force required to maintain a constant walkway speed.



4– A bowling ball (a sphere of radius R with uniform density) hits the floor with an initial angular velocity ω_0 as shown in the figure. The coefficient of kinetic friction bteween the ball and the floor is μ_k. What is the final velocity of the center of mass of the bowling ball?



5- A horizontal platform rotates about on low-friction bearings about its center axis with an angular velocity of ω_0. Its moment of inertia about the center axis is I_0. A small object of mass m is placed on the platform at a distance r_1 form the rotation axis with a velocity v as shown in the figure. The object initially slides but it eventually corotates with the platform when it is at a distance r_2. Find the final angular velocity of the platform in terms of the parameters given in the problem.



6- Evidence for dark matter comes from "flat" rotation curves of galaxies. Show that if the dark matter density in the halo follows a spherically symmetric distribution $\rho(r) \propto r^{-2}$, then the circular velocity $v(r)$ of the luminous matter is a constant. Assume that the motion of the luminous matter is purely due to the gravity of the dark matter.

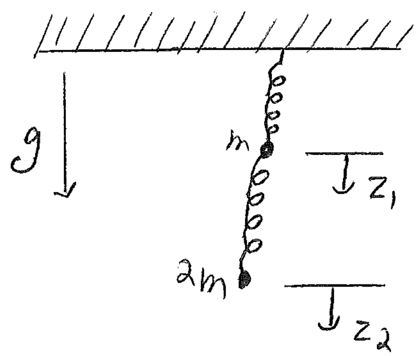
7– Consider an attractive central force $F(r) = -k/r^4$ where r is the distance to the origin. Show that circulart orbits are unstable for this force.

8- A thin homogeneous plate lies in the $y - z$ plane. Its moment of inertia tensor in the xyz basis is given by

$$I = I_0 \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & -3 & 2 \end{pmatrix},$$

where I_0 is a constant. If the plate rotates about the z axis with a constant angular velocity ω, what torque must be applied to it to maintain its motion?

9– Two masses m and $2m$ are attached to each other by a massless spring with spring constant k and suspended from the ceiling by an identical spring as shown in the figure. Use coordinates z_1 and z_2 to describe the vertical displacements of the upper and lower masses from the equilibrium positions, respectively, and write the equations of motion for the two masses. Find the normal mode frequencies of this system.



10– A particle with mass m is constrained to move on the surface of a circular cone of half-angle α with its axis on the vertical z axis and vertex at the origin. The particle moves without friction. Write down a Lagrangian L in terms of the spherical coordinates and find the equations of motion for r and ϕ.