**Preliminary Examination: Mechanics**

*Department of Physics and Astronomy*

*University of New Mexico*

**Fall Semester 2013**

**Instructions:**

- *sign in with your name and ID; put ID only on exam!*
- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* $8 \times 11$ *page are allowed; notes must be handed in with exam!*
- *total time is 3 hours*

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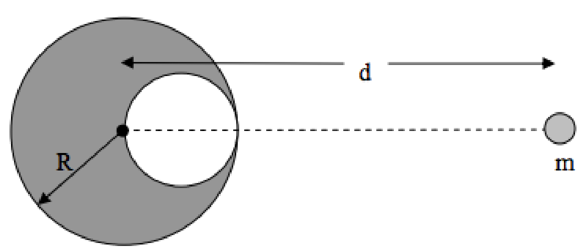
1. A toy rocket has an engine that loses mass linearly with time: $dm/dt = -k$ with k a positive constant. The relative velocity U between a rocket and its ejected exhaust gas is a constant. The initial mass of the rocket is $m_0$. Since it is a toy, it doesn't go very high so the force of gravity remains constant. Find the minimum exhaust speed U for the rocket to take off immediately when fired. Calculate the velocity of the rocket as a function of time. Show that for sufficiently small time the velocity increases linearly and find the proportionality constant.

2. The figure below shows a spherical hollow inside an otherwise solid lead sphere of radius R. The surface of the hollow passes through the center of the sphere and touches the right side of the sphere. The mass of the sphere before hollowing was M. With what gravitational force does the hollowed out lead sphere attract a small sphere of mass m that lies at a distance d from the center of the lead sphere on a straight line connecting the centers of the sphere and the hollow?

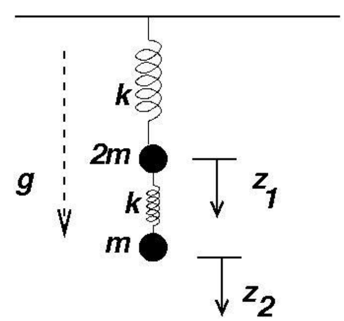


3. Two masses $2m$ and $m$ are attached to each other by a massless spring with spring constant $k$ and and suspended from the ceiling by an identical spring (refer to figure). Only vertical displacements are considered. Using coordinates $z_1$ and $z_2$ to describe the displacements from equilibrium of the upper and lower masses respectively, the equations of motion are given by,

$$2m\ddot{z}_1 + 2kz_1 = kz_2$$

$$m\ddot{z}_2 + kz_2 = kz_1$$

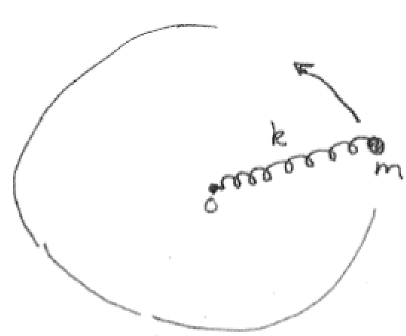
Determine the normal modes and normal mode frequencies of this system.



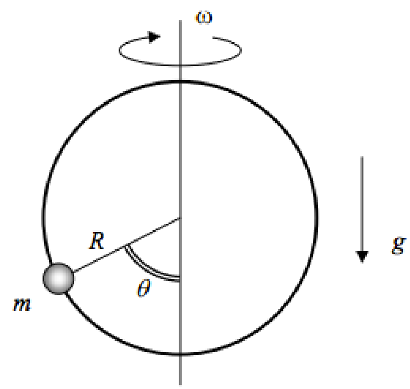

4. A mass m moving in a circular orbit about the origin is attracted by a three dimensional harmonic potential,

$$U(r) = \frac{1}{2}kr^2$$

Show that the angular frequency $\omega$ is independent of r. Write down the Lagrangian and find the radial equation of motion. If a small kick is applied in the radial direction, show that the frequency of small oscillations in the radial direction is $2\omega$.

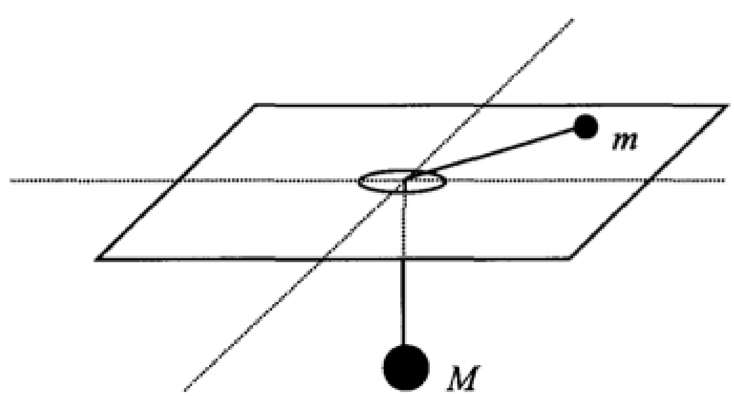


5. A bead may slide, without friction, on a circular hoop of radius R that is rotated about its vertical diameter with a constant angular velocity $\omega$ (see Fig. below). Taking into account the uniform gravity field write down the Lagrangian taking the angle $\theta$ of bead deviation from the lowest point as the generalized coordinate.



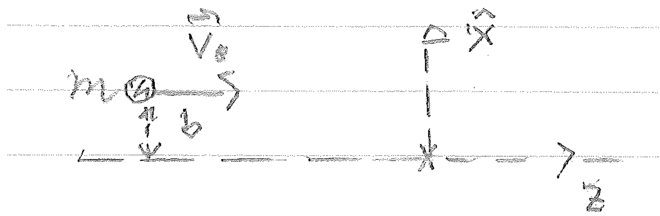

6. A mass $m$ moves on a frictionless table. It is tied to a string which runs through a hole in the table. A mass $M$ hangs from the other end ofthe string and acted upon by gravity. $M$ is constrained to move vertically.

   (a) Choose appropriate coordinates and write a Lagrangian for the system.
   (b) Describe the symmetries and conserved quantities.
   (c) Find the solutions for which the height of mass M is constant.

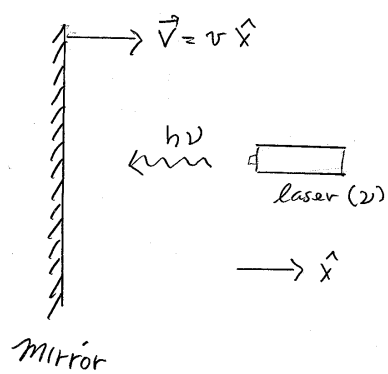


7. A neutron of kinetic energy 2 MeV collides head-on in an elastic collision with a nuclei of mass number A which is initially at rest. What is the smallest energy the neutron can have following the collision?

8. A particle of mass m collides with a fixed force center that acts on the mass with a repulsive force of $k/r^2$. It has initial velocity $v_0$ in the z direction and impact parameter b. Find the distance of closest approach of the mass.




9. A laser with frequency $\nu$ is aimed at a mirror which moves towards it at a relativistic speed $v$. Use the Lorentz transformation to find the frequency $\nu'$ of photons that are reflected directly back towards the laser (in the rest frame of the laser).



10. A cord of length $\ell$ and uniform mass density $\lambda$ is attached to the ceiling by one end, at $x = 0$. The other end passes through a tiny hole in a rigid plate, directly below at $x = \ell$ as shown in the Figure. A mass $M$ hangs from the lower end of the cord in order to keep it taut. Write down an expression that describes the tension in the cord as a function of $x$. Suppose that at a time $t = 0$ the cord is given a small perpendicular displacement $u(x, 0)$ along its length. Show that the linear partial differential equation,

$$\lambda \frac{\partial^2 u}{\partial t^2} = g \left[ M + \lambda \left( \ell - x \right) \right] \frac{\partial^2 u}{\partial x^2} - g\lambda \frac{\partial u}{\partial x}$$

describes the subsequent evolution of the transverse wave $u(x, t)$.