Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico

Spring Semester 2011

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours*

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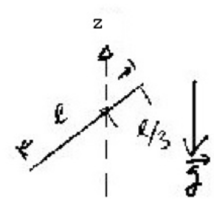
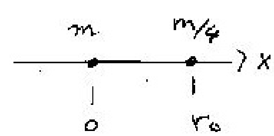
1. Consider a long thin rod of uniform density with length ℓ, mass m and negligible diameter. The rod is pivoted about $\ell = 1/3$ and free to move in the vertical plane under the force of gravity. Find the frequency of small oscillations for the pendulum.



2. A mass moves in one dimension (call it x) subject to a viscous drag force F_d. It starts at $x = 0$ with speed v_0. Take $F_d = -b\dot{x}$, show that the mass does not come to rest in any finite time. Find the limiting distance the mass moves.

3. Two masses a, b interact with an attractive central force $F = -kr$ where r is the separation of the masses. There are no other forces acting. Mass $m_a = m$, $m_b = m/4$ and both masses are initially at rested separated by a distance r_0. Some time later they collide. *Define the positive x axis as starting from m_a at $x = 0$ and going through m_b at $x = r_0$ at $t = 0$.*
(a) Where do they collide?
(b) What is their relative velocity just before they collide?



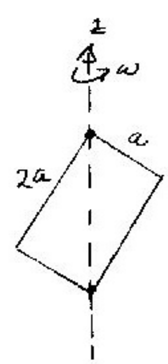
4. Evidence for dark matter comes from "flat" rotation curves of galaxies. Assume

that the observed matter moves in circular orbits about the center of the galaxy and that the velocity of the matter as a function of the radius $v(r)$ is a constant. Also assume the motion of the observed matter is purely due to the gravity of the dark matter (mass of luminous matter is negligible) and the dark matter is distributed with spherical symmetry about the center of the galaxy. What is the density $\rho(r)$ of the dark matter as a function of radius?

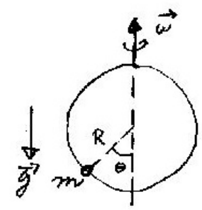
5. a) Derive Kepler's second ("equal areas in equal times") law.
   b) Derive Kepler's third (period^2 $\propto$ semi-major axis^3) law for the special case of circular orbits.

6. The principle of the synchrotron proton accelerator is to keep the beam radius constant by increasing the strength of the magnetic field with increasing particle momentum. In order to accomplish this, the beam frequency must be known as a function of proton momentum. Derive this relation. Neglect proton synchrotron radiation. Check that in the limit of $p >> mc$ the frequency becomes a constant.

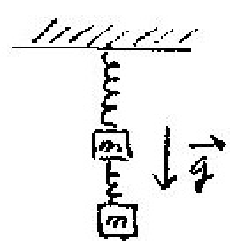
7. A door of uniform mass density and width a and height 2a is rotated about a vertical axle through two diagonal corners. The bearings supporting the plate are mounted just at the corners. If it is then rotated at constant angular frequency ω, find the torque that must be supplied by the bearings.



8. Consider a bead of mass m constrained to move on a circular wire hoop of radius R. The hoop lies in the vertical plane and is rotated at constant angular velocity ω about an axis through the center of the hoop. The bead slides without friction on the hoop and is acted on by the force of gravity. Use the coordinate θ to measure the polar angle from the bottom of the hoop, and the angle ϕ to measure the azimuthal angle ($\dot{\phi} = \omega$). Write the Lagrangian for the system and find the equation of motion for θ. Find the critical value of ω such that for $\omega < \omega_c$ the point $\theta = 0$ is a stable equilibrium.



9. Two masses are suspended by massless springs, one from the other in the vertical direction. Both masses have mass m and both springs have constants k and un-stretched length ℓ_0. Find the normal modes and frequencies of oscillation for the system.



10. Two pucks of mass m side freely on a horizontal plane. They are connected by a spring (constant k and negligible un-stretched length) and set in circular motion with angular momentum L. The pucks are given a small, simultaneous radial poke. What is the frequency of subsequent radial oscillations?