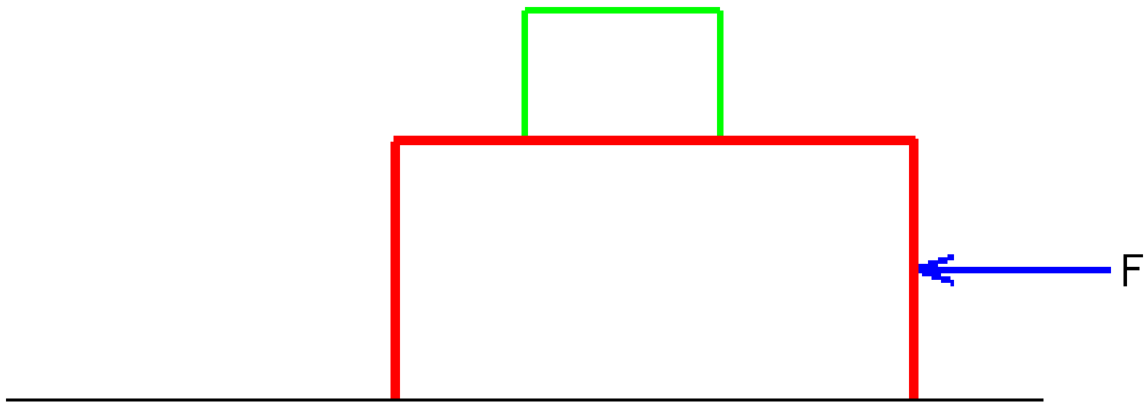
Preliminary Examination: Classical Mechanics
Department of Physics and Astronomy
University of New Mexico
Spring, 2009

Instructions:

- The exam consists of 10 problems, 10 points each.
- Where possible, show all work; partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2} \times 11$ page are allowed.
- Total time is 3 hours.

1. A large rectangular block of wood, with mass $M = 10$ kg, is being pushed across a horizontal, frictionless floor by a horizontal force of magnitude $F = 24$ Newtons. On top of the block of wood is a smaller object, of mass $m = 2$ kg, with a fairly slippery surface between the two, which can be characterized by saying that the coefficient of sliding friction between them is only $\mu = 0.1$ What is the direction and magnitude of the force of friction between the two objects, and what is the relative acceleration of the two objects?



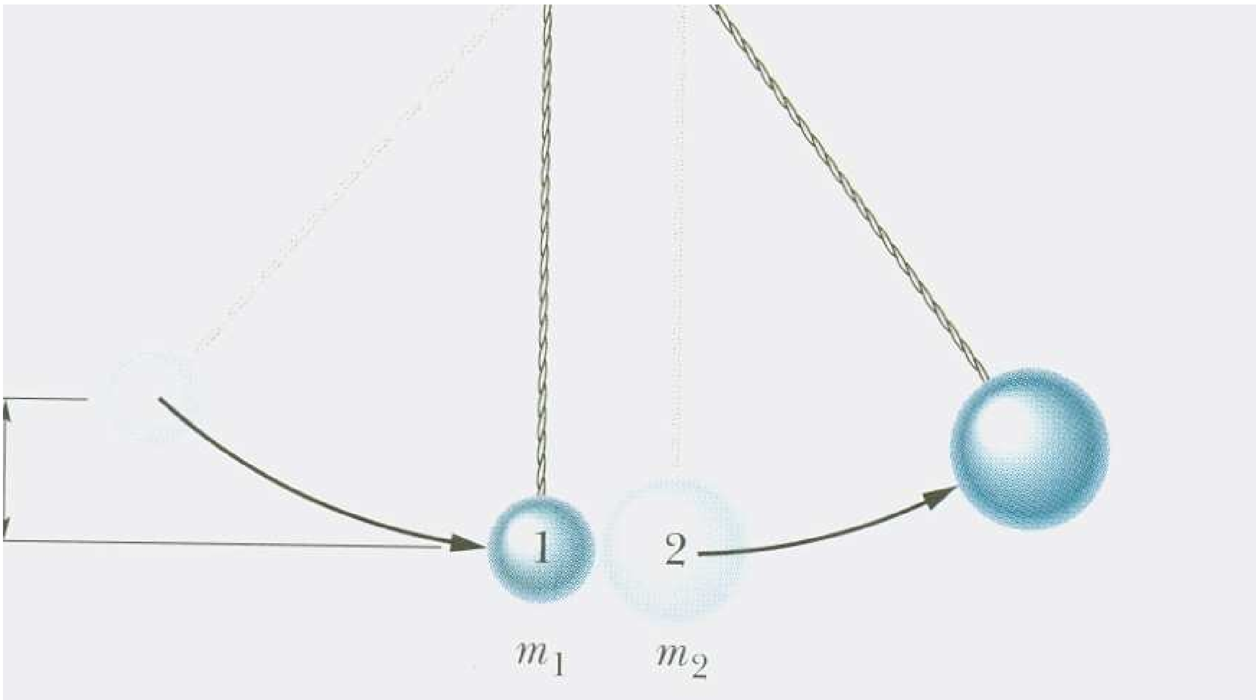
2. The United Nations Air Force has just deflected an asteroid that was headed toward collision with the Earth. Its course has been changed so that now it is headed directly toward the Sun, with a speed of 5 km/sec, as it leaves the orbit of the Earth. When it arrives at the orbit of Venus, what will be its speed. Take the radii of the orbits of Earth and Venus to be 1 AU and 0.72 AU, and the gravitational "pull" of the sun, namely $GM_{\text{sun}} = (2\pi)^2$ AU^3/yr^2, while the conversion factor 1 AU/yr = 4.74 km/sec would probably be useful.

3. A damped harmonic oscillator could be described by some single coordinate, say $x = x(t)$, which would satisfy the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \ .$$

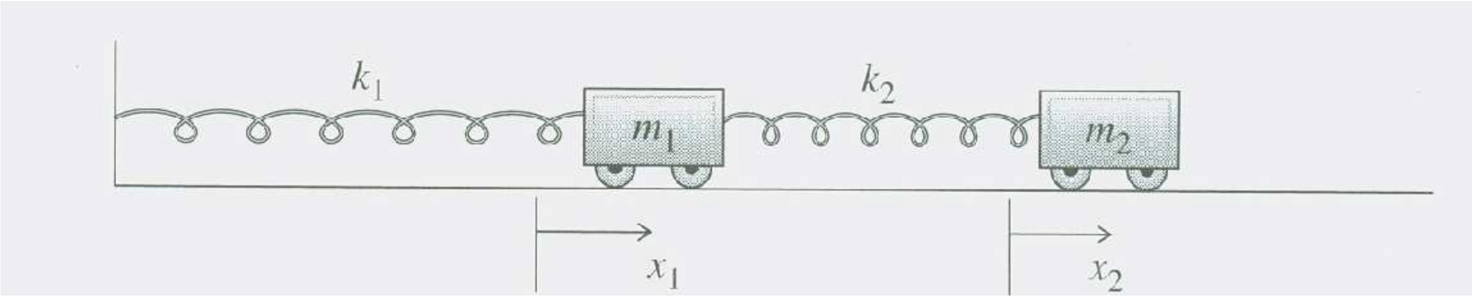
A particular such oscillator has period $\tau_0 = 1.000$ seconds when there is no damping. However, when some particular amount of damping is added to it the period changes to $\tau_1 = 1.001$ seconds. (Note that we define the period as the time between successive maxima of $x(t)$.) Calculate the damping factor, β, and determine the decrease in the amplitude of oscillation after 10 cycles.

4. Two metal spheres are suspended by cords hanging from the ceiling, so that they are just touching when they are at rest, and hanging vertically downward. The one sphere has mass $m_1 = 2$ kg, while the other one has mass $m_2 = 3$ kg. The smaller sphere is swung to one side, to a (vertical) height of 10 cm above where it was hanging before, and then released. What is the velocity of the other sphere just after they collide? Assuming the collision was completely elastic, to what height will the second sphere rise?



5. A stationary space station can be approximated as a hollow spherical shell, with mass $m = 10^4$ kg, and inner and outer radii of 8 meters and 9 meters. To change the orientation of the station, a uniform solid disk used as a flywheel (of radius 10 cm and mass 10 kg) is located at the center of the ship, and is spun up quickly from rest to 1000 rpm. How long will it take the station to rotate by 10 degrees? How much energy will be needed for the entire operation? [Note that the moment of inertia of a solid sphere is $\frac{2}{5}mR^2$.]

6. Two carts are shown, of equal masses. They are connected by a spring, and the left-hand one is connected by an identical spring to the wall. Find and describe each of the normal frequencies of oscillation, and sketch all the motions.



7. A certain mass, m, is subject to gravity and other forces in such a way that its equation of motion may be given as

$$2m\ddot{r} = \frac{\ell^2}{mr^3} - mg \ ,$$

where g and ℓ are constants.

a. Determine an equilibrium position, r_0, for the coordinate r for this mass, as a function of ℓ, m and g.

b. If the mass is put at a point near that equilibrium position, so that $r = r_0 + \epsilon$, it will oscillate about the equilibrium position for small values of ϵ. What will be the frequency of that oscillation?

8. Near the surface of our rotating earth, a moving object is subject to centrifugal and Coriolis forces as well as the usual gravitational force. We propose to consider the motion of a ship on the ocean at North latitude 30°, where we can write out Newton's equations as

$$\ddot{\vec{r}} = 2\dot{\vec{r}} \times \vec{\Omega} \ ,$$

where $\vec{\Omega}$ is the angular velocity of the earth. Taking a local set of basis vectors, as used in the ship's cabin, which we may describe as "East," "North," and "Up," what is the form of $\vec{\Omega}$, and what is the magnitude and direction of the Coriolis acceleration of the ship, if its velocity is 5 m/sec in the eastward direction?

9. A particular system with one degree of freedom has the Hamiltonian given by

$$\mathcal{H} = \tfrac{1}{2}p^2 + p \sin\psi \ ,$$

where the variable ψ is an angle, so that it is restricted to lie between $-\pi$ and $+\pi$, and the conjugate momentum p has been normalized to be dimensionless. Find the (Hamiltonian) equations of motion for the system, the Lagrangian, which is $\mathcal{L}(\psi, \dot{\psi})$, and the Lagrangian equations of motion.

10. Consider a bead of mass m sliding without friction on a wire that is bent in the shape of a parabola, and is being spun with constant velocity ω about its vertical axis, as shown in the figure. Use cylindrical coordinates, $\{\rho, \phi, z\}$, and let the equation of the parabola be $z = k\rho^2$. Write down the Lagrangian in terms of ρ as the generalized coordinate. Find the equation of motion of the bead, and determine whether there are positions of equilibrium. Discuss their stability.