Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico
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Instructions:

- The exam consists of 10 problems, 10 points each;
- Partial credit will be given if merited;
- Personal notes on two sides of an 8 x 11 page are allowed;
- Total time is 3 hours.

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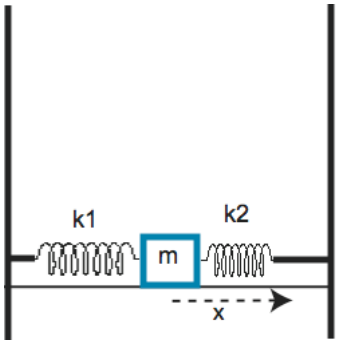
1. A king commissions two crowns. They are supposed to be identical and when weighed have identical mass, but the king suspects that one of them is only gold plated and not solid gold. Describe a non-destructive experiment to determine if the two crowns are identical in composition.

2. A car traveling at 30 kph goes into a corner with a radius of curvature of 150 m. If the road is icy (no friction), what angle (in degrees) does the road have to be banked at in order for the car not to go into the ditch on either side of the road?

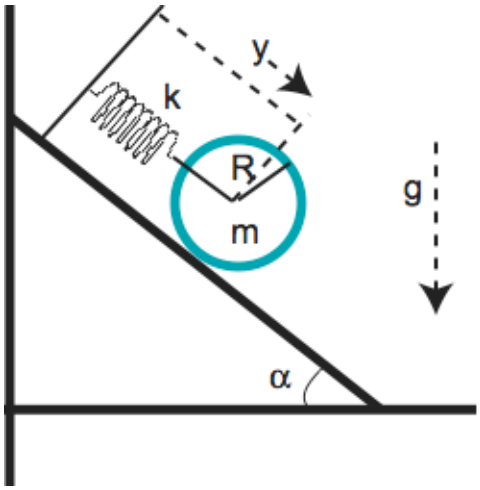
3. Consider a moon of mass m going around a planet of mass M and radius R. with some angular velocity, ω. The orbital radius for the moon is r. Show that if tidal forces act to slow down the rotation of the planet, the orbital radius, r, for the moon must increase. You can ignore the rotation of the moon. Hint: the moment of inertia for a sphere is $I = 2/5\ MR^2$.

4. Consider a mass attached to two walls by springs of constants k1 and k2 (see figure), and sitting on a frictionless surface. Derive the frequency of oscillation ω for a displacement in the direction of compression of either spring (i.e., along the x axis).



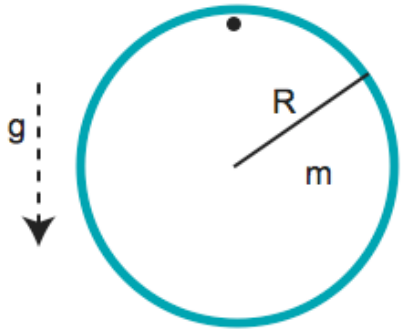
5. A galaxy is observed to have the Hydrogen Hα line (rest wavelength 6563 angstroms) at 7111 angstroms. What is its relative velocity to us in km/s? Is the galaxy approaching or receding?

6. A hoop of mass, *m*, and radius *R* rolls without slipping down a plane as shown in the figure below. A massless spring is attached to a massless axle. Write a Lagrangian for the system and use it to derive the equation of motion in the y coordinate.



7. Show that for a planet in a circular orbit around a star, the velocity needed for the planet to escape from the star is just sqrt(2) times the orbital velocity.

8. A hula hoop of mass, m, and radius R, is suspended on a nail and is free to pivot about that point. Find the period of oscillation for small (harmonic) oscillations. You can ignore the thickness of the hoop.



9. Starting with Kepler's 2nd law which can be written as $dA/dt = L/2m$ where A is the area swept out by the orbit of an object of mass, m, orbiting a body of mass M, and L is the angular momentum, derive the period of rotation, P, as a function of the radius r, assuming a circular orbit. Show that this is a simplified form of Kepler's 3rd law.

10. A free neutron at rest decays into a proton and an electron and an electron anti-neutrino. The rest masses for a neutron, a proton, and an electron are, m_n = 939.565 MeV/c^2, m_p = 938.272 MeV/c^2, and m_e = 0.511 MeV/c^2. Assuming that for the moment we ignore the anti-neutrino, find (a) the total energy released by the decay; (b) the kinetic energy carried away by the proton and (c) the kinetic energy carried away by the electron. Justify any approximations you make to obtain a solution.