# Preliminary Examination: Mechanics

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*University of New Mexico*
**Spring 2007**

**Instructions:**

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* $8 \times 11$ *page are allowed;*
- *total time is 3 hours.*

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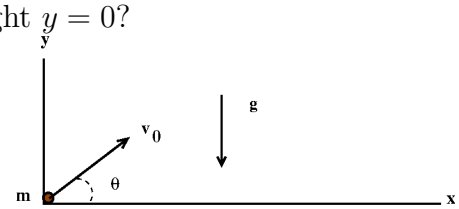
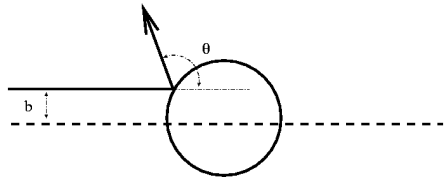
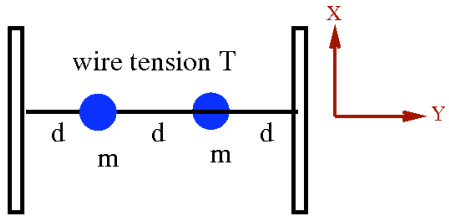
1. A ball of mass $m$ is thrown from the origin with initial velocity $v_0$ and inclination angle $\theta$ as shown in the figure (note gravity acts in the $-y$ direction). Neglecting air-friction, what is the range of the projectile (distance it travels in the x direction when it returns to height $y = 0$?



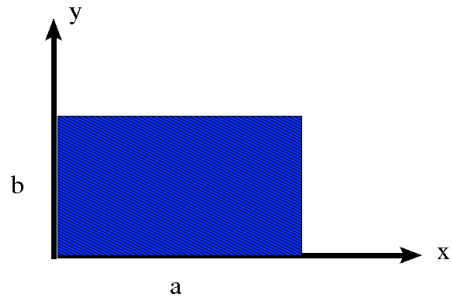

2. Consider a spherical, non-rotating planet of uniform density with radius $R$ and mass $M$. A straight hole is drilled through the center of the planet. If a small test mass is dropped through the hole, show that in the absence of friction the test mass will undergo simple harmonic motion and find the period of the motion in terms of the given parameters and Newton's gravitational constant $G$.
3. Consider the central-force motion of a planet about the sun. Show that conservation of angular momentum implies Kepler's second law, "Equal areas in equal time".
4. Consider the motion of two point-particles having masses $m_1$ and $m_2$ and attracted to one another by a central force. Write a Lagrangian for the system in terms of coordinates for $m_1$ and $m_2$ with respect to a fixed origin. Define coordinates for the center of mass and relative motions, and show that your Lagrangian separates. Prove that the total momentum is conserved.
5. For classical scattering from a hard sphere of radius R, the relation between impact parameter $b$ and the scattering angle $\theta$ is $b = R\cos(\theta/2)$. Find the differential cross section.



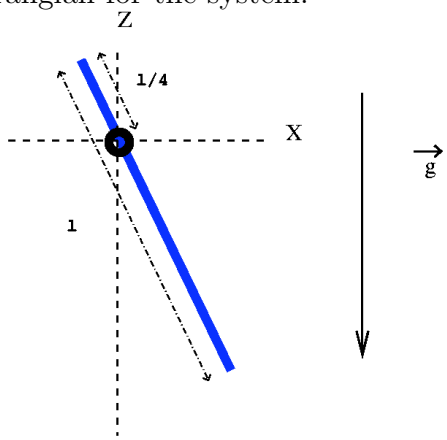
6. Two beads of mass $m$ are attached to a wire of negligible mass stretched between fixed endpoints . Find the equations of motion, normal modes, and freqencies of oscillations for small displacements from equilibrium in the x direction. The masses are equally spaced along the wire as shown in the figure and the tension in the wire is $T$.




7. A nucleus of mass $M$ is initial at rest. It undergoes $\alpha$ decay to a daughter nucleus of mass $M_D > m_\alpha$. The decay releases an energy $Q << m_\alpha c^2$. Assuming all of this energy is released as kinetic energy of the daughter and the $\alpha$ particle, determine the kinetic energy of the $\alpha$ particle.
8. Consider a uniform rectangular flat plate with mass $m$ and sides of length $a, b$. The thickness of the plate is negligible. The plate lies in the $x-y$ plane as shown in the figure. Consider the moments of the inertia tensor with respect to the given coordinate system. Find the moments of the inertia tensor in the $x-y$ plane: $I_{xx}$, $I_{xy}$, $I_{yx}$, $I_{yy}$




9. A uniform rigid rod of length $\ell$ is pivoted about a horizontal axis at a distance $\ell/4$ from the end so that it is free to rotate in the vertical plane under the force of gravity. Write a Lagrangian for the system.




10. Prove that the absorption of a photon by a free electron ($e + \gamma \rightarrow e$) cannot occur.