Preliminary Examination: Mechanics

Department of Physics and Astronomy

University of New Mexico

Spring 2006

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours*

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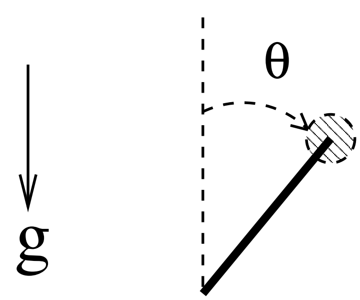
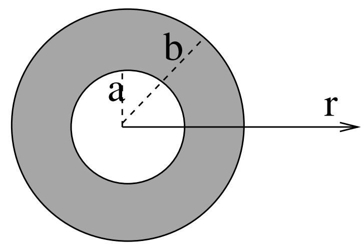
1. In a freshman physics demonstration, two carts are held at rest on an air track with a compressed spring between them. When the carts are released, the spring releases an energy E_0. If carts one and two have masses of m_1, m_2 respectively, what is the speed of cart one?

2. Consider a system of two identical masses (m) connected by a spring (constant k) which is constrained to move along the direction connecting the two masses, but can move freely along that direction. At $t = 0$ mass 1 is at rest and mass two has a velocity $\vec{v} = v_2(0)\hat{x}$ and the spring at its unstretched length. What is the velocity of mass 1 as a function of time t for $t > 0$?

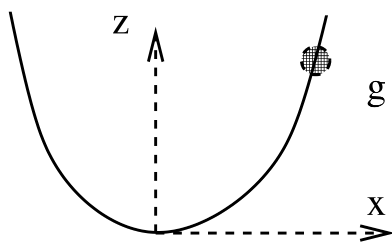
3. A mass attached to a rigid rod of length ℓ rotates in a vertical plane with constant angular speed ω. What is the magnitude of the force that must be applied to the mass by the rod as a function of angle θ? Take $\theta = 0$ to be the vertical direction.



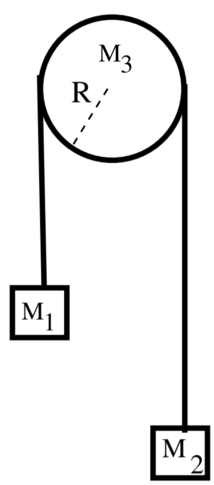
4. Consider a uniform density spherical shell of total mass M and inner radius a, outer radius b. Calculate the force on a test particle (mass m_t) everywhere from $0 < r < \infty$.



5. A particle of mass m slides without friction on a bent wire subject to the force of gravity. Taking the vertical direction to be $\hat{z}$ and the horizontal direction to be $\hat{x}$, the shape of the wire is given by $z = ax^2/2$. Write the Lagrangian taking x as the generalized coordinate.



6. Consider an Atwood machine with masses m_1, m_2 and with pully of mass m_3 and radius R. Find the equation of motion for m_1.



7. A simple pendulum (mass m, length l) is emersed in a viscous fluid, giving rise to a resistive force proportional to the velocity of the pendulum bob,

$$\vec{F}_{res} = -2m\beta\vec{v}$$

Show that the motion of the system valid for small angles is of the form,

$$\theta(t) = e^{-\beta t}(Ae^{i\Omega t} + Be^{-i\Omega t}),$$

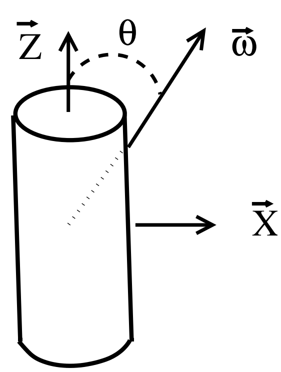
For what values of the parameters will the pendulum be lightly damped? For light damping, what is $\theta(t)$ if the pendulum is released from rest at angle θ_0?

8. A particle of mass m is bound to the origin by a force $\vec{F} = -k\hat{r}$. The Lagrangian of the system may therefore be taken to be,

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{1}{2}kr^2.$$

Show that circular orbits are stable.

9. A cylinder of radius r and height ℓ is rotated about its center of mass with fixed angular velocity $\vec{\omega}$. The angular velocity $\vec{\omega}$ makes an angle θ with respect to the cylinder's axis (see figure). The moment of inertia tensor for the cylinder in body-centered coordinates is diagonal with eigenvalues $I_{xx} = I_{yy} = \frac{1}{12}m\ell^2$, $I_{zz} = \frac{1}{2}mr^2$. What is the magnitude of the torque required to keep the cylinder rotating with this angular velocity?



10. A particle called a neutral pion π_0 (rest energy $m_\pi c^2$) decays to two photons. Suppose the π_0 has some initial momentum in the lab $\vec{p} = p\hat{x}$ and one photon is detected in the $+x$ direction with energy E_+ and the other in the $-x$ direction with energy E_-. From these observations, what is the speed of the pion in the lab?