Department of Physics and Astronomy, University of New Mexico

Classical Mechanics Preliminary Examination

Fall 2018

Instructions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- NO cheat sheets are allowed.
- Total time: 3 hours.

Useful Constants, Formulas, and Relations:

- Mass of the Earth: $M_{\rm E} = 6 \times 10^{24}$kg, Radius of the Earth: $R_{\rm E} = 6.4 \times 10^{3}$km
- Gravitational constant: $G = 6.7 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
- Moment of inertial of a uniform solid sphere of mass M and radius R: $I = \frac{2}{5}MR^2$
- Moment of inertia of a uniform rectangular plate of mass M, length l, and width w situated in the xy plane about its principal axes:

$$\begin{pmatrix} \frac{1}{12}Ml^2 & 0 & 0 \\ 0 & \frac{1}{12}Mw^2 & 0 \\ 0 & 0 & \frac{1}{12}M(l^2+w^2) \end{pmatrix}$$

- Euler's equation for a rotating rigid body:

$$\frac{d\vec{L}}{dt} = \frac{\partial \vec{L}}{\partial t} + \vec{\omega} \times \vec{L}$$

- The relation between the energy E and momentum p of an object with mass m ($p \equiv |\vec{p}|$):

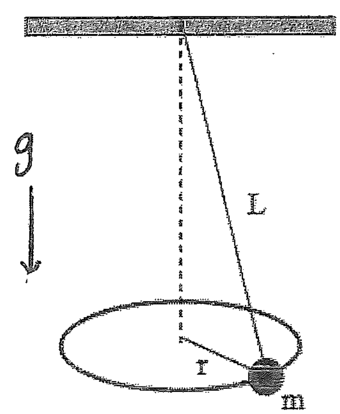
$$E^2 = (pc)^2 + (mc^2)^2$$

1- Consider a hypoethetical mine shaft that extends form the north pole to center of the Earth. If a rock is released from rest over the shaft at the north pole, what will be its speed at the center of the Earth? Assume uniform density for the Earth and neglect friction.

2- A spaceship is hovering at a constant height just above the surface of a planet where the acceleration of gravity is g. The speed of exahust u is constant, and the initial mass of the spaceship is m_0. How long will it take until the mass of the spaceship is reduced to half of its initial value?

3- A damped harmonic oscillator consists of a mass m that is connected to a wall by a spring with spring constant k in a viscous medium with damping coefficient b (assume damping is porportional to the velocity). The oscillator is driven by a periodic external force $F(t) = F_0 \sin(\omega t)$. Derive an expression for the position of the mass in steady state, and find the amplitude of oscillations at resonance.

4- A bob with mass m that is connected to the ceiling by a massless string of length L follows a circular path as shown in the figure. Find the tension in the string T and the frequency of motion f.



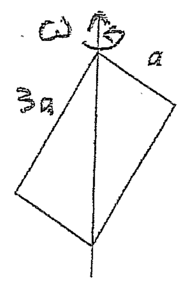
5- A billiard ball is hit by a cue and starts rolling without slipping. Assuming that the cue delivers a horizontal impulse, find the height at which the ball should be struck.

6- A satellite with mass m orbits the Earth. The Earth-satellite distance r and the radial velocity $\dot{r}$ are related according to:

$$E = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} - \frac{GmM_{\mathrm{E}}}{r},$$

where E is the energy and l is the angular momentum. Find the relation between E and l so that the aphelion (r_{max}) is two times larger than the prehelion (r_{min}).

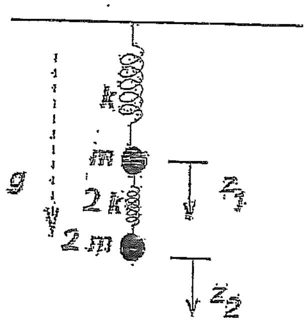
7- A door of mass M, width a and height $3a$ is rotated about a vertical axle as shown below. The bearings supporting the plate are mounted just at the corners. Find the torque that must be supplied in order for the angular velocity to be constant at the instant shown.



8- Two masses m and $2m$ are attached to each other by a massless spring with spring constant k and and suspended from the ceiling by another spring with spring constant $2k$ as shown in figure. Using coordinates z_1 and z_2 to describe the vertical displacements from thei equilibrium positions of the upper and lower masses respectively, show that the equations of motion are given by:

$$m\ddot{z}_1 + 3kz_1 - 2kz_2 = 0$$
$$m\ddot{z}_2 + kz_2 - kz_1 = 0\,.$$

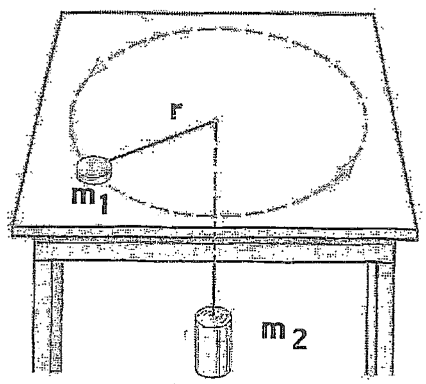
Determine the normal modes and the corresponding frequencies of this system.



9- A mass m_1 moves on a frictionless table. It is connected to a second mass m_2 by a massless string that runs through a hole in the table as shown in the figure. m_2 is constrained to move vertically. By choosing appropriate coordinates, write a Lagrangian for this system and derive the radial equation of motion:

$$(m_1 + m_2)\ddot{r} = \frac{l^2}{m_1 r^3} - m_2 g\,,$$

where l is the angular momentum of m_1.



10- A photon with energy E_γ undergoes head-on collision with an electron at rest. Find the energy of the photon E'_γ after the collision.