Department of Physics and Astronomy, University of New Mexico

Classical Mechanics Preliminary Examination

Fall 2017

Instructions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- NO cheat sheets are allowed.
- Total time: 3 hours.

Useful Constants, Formulas, and Relations:

- Mass of the earth: $M_E \approx 6 \times 10^{24}$ kg, Radius of the earth: $R_E \approx 6400$ km
- Gravitational constant: $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
- Moment of inertial of a uniform solid sphere of mass M and radius R: $I = \frac{2}{5}MR^2$
- Moment of inertial of a uniform rod of mass M and length l about its center of mass: $I = \frac{1}{12}Ml^2$
- Moment of inertia of a uniform rectangular plate of mass M, length l, and width w that is situated in the xy plane about its principal axes:

$$\begin{pmatrix} \frac{1}{12}Ml^2 & 0 & 0 \\ 0 & \frac{1}{12}Mw^2 & 0 \\ 0 & 0 & \frac{1}{12}M(l^2+w^2) \end{pmatrix}$$

- Euler's equation for a rotating rigid body:

$$\frac{d\vec{L}}{dt} = \frac{\partial \vec{L}}{\partial t} + \vec{\omega} \times \vec{L}$$

- The relation between the energy E and momentum p of an object with mass m ($p \equiv |\vec{p}|$):

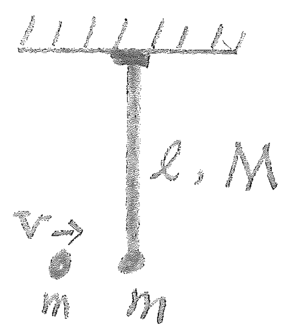
$$E^2 = (pc)^2 + (mc^2)^2$$

1– Earth moves around the sun in an approximately circular orbit of radius $R \approx 1.50 \times 10^8$ km. Estimate mass of the sun.

2- A rock of mass m is at the bottom of a super-deep mine shaft that is 1/2 radius of the earth. Ignoring friction, find the minimum speed to launch the rock so that it can reach the surface of the earth. Make any simplifying assumption(s) that you need.

3– A spacecraft with initial mass m_0 is coasting with initial speed V_0 when it enters a very dilute dust wind that has a uniform mass density ρ and moves with a constant speed v in the opposite direction. Assume that the collision cross-sectional area A for gathering dust remains constant and that all dust encountered sticks to the spacecraft. What is the spacecrafts speed when its mass reaches $2m_0$?

4– A mass m hits an identical ball that is hanging from the ceiling by a uniform rigid rod of length l and mass M (the rod is hinged to the ceiling). The two balls stick together after the collision. Find the minimum speed of the ball before the collision such that the system reaches the ceiling.

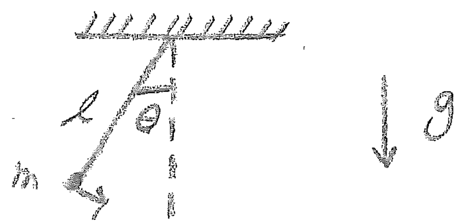


5- Find the height above the center of a billiard ball of mass M and radius R at which the ball should be struck so that it will roll with no initial slipping. Assume that the cue delivers a horizontal impulse to the ball.

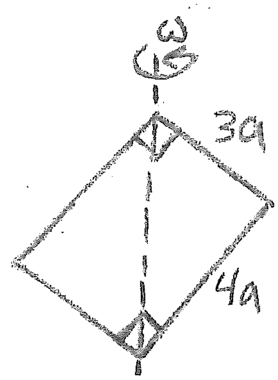
6- A simple pendulum is made of a rigid, massless rod of length l at the end of which a point-mass m hangs. The pendulum moves at small anglein the vertical plane and experiences a damping force

$$\vec{F} = -\beta\dot{\theta}\hat{\theta}\,,$$

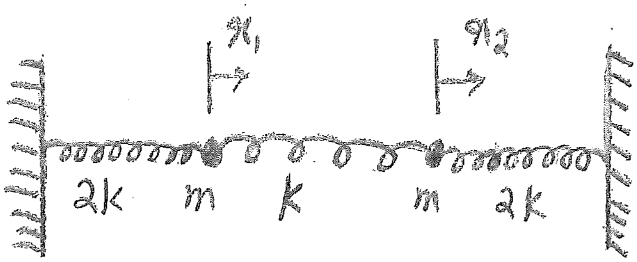
where β is a positive constant. Find the maximum value of β to have an oscillatory motion.



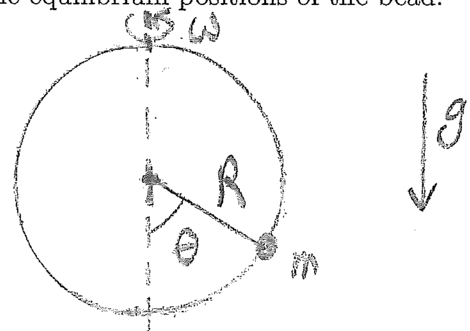
7– A uniform rigid rectangle of mass M is rotated about one of its diameters as shown in the figure. Find the torque that must be supplied in order for the angular velocity to be constant at the instant shown.



8- Two identical masses m are connected to each other and to two walls by three springs as shown in the figure. Using the displacements x_1 and x_2 of the masses from theor equilibrium positions, write the equations of motion for the tweo masses. Find the normal modes and their corresponding frequencies of this system.



9– A bead of mass m can slide without friction on a circular hoop of radius R that is rotated about its vertical diameter with a constant angular velocity ω. Write down a Lagrangian in terms of angle θ shown below and find the equation of motion of the bead. Assuming that that $\omega^2 > g/R$, find the equilibrium positions of the bead.



10– A neutral pion π_0 decay to two photons. Suppose that π_0 has an initial velocity v in the $+x$ direction, and that the photons are emitted in the $+x$ direction and $-x$ direction respectively. Find the respective energies of the photons E_+ and E_- in terms of the pion mass m_π and its velocity v.