Department of Physics and Astronomy, University of New Mexico

Classical Mechanics Preliminary Examination

Fall 2016

In structions:

- You should attempt all 10 problems (10 points each).
- Partial credit will be given if merited.
- NO cheat sheets are allowed.
- Total time: 3 hours.

Useful Constants, Formulas, and Relations

- Mass of the earth: $M_E = 5.97 \times 10^{24}$kg
- Gravitational constant: $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
- Moment of inertial of a uniform solid sphere of mass M and radius R: $I = \frac{2}{5}MR^2$
- Euler's equation for a rotating rigid body:

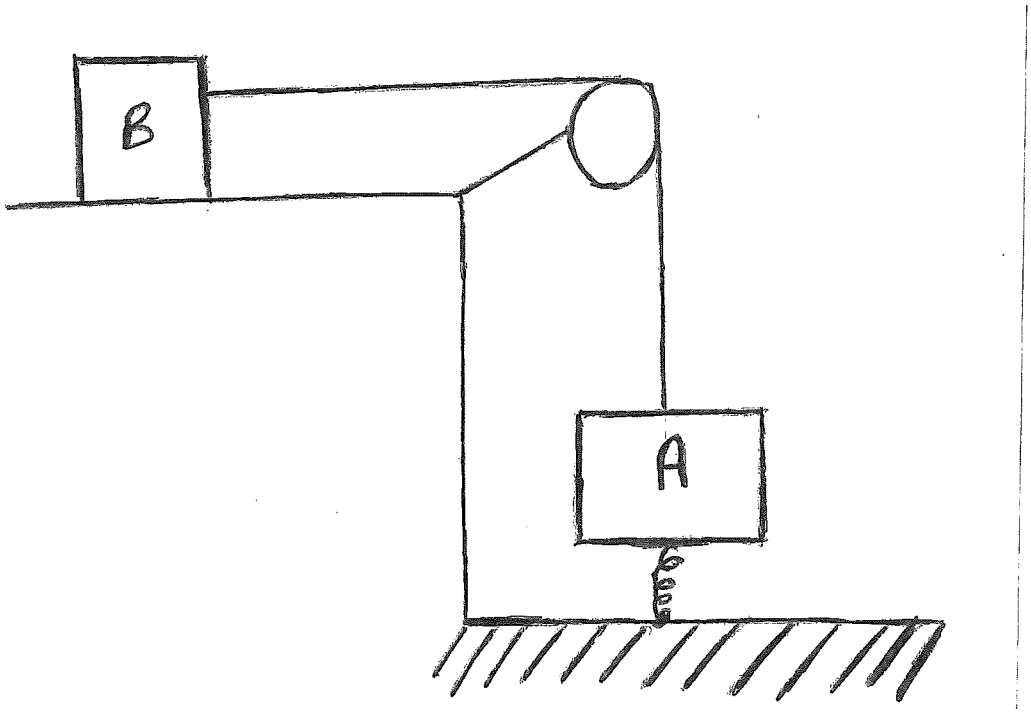
$$\frac{d\vec{L}}{dt} = \frac{\partial \vec{L}}{\partial t} + \vec{\omega} \times \vec{L}$$

- The relation between the energy E and momentum p of an object with mass m ($p \equiv |\vec{p}|$):

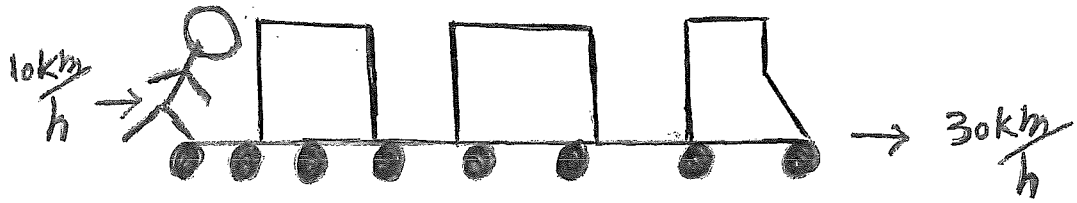
$$E^2 = (pc)^2 + (mc^2)^2$$

1– Moon moves around the earth in an approximately circular orbit. Using the orbital priod of $T = 28$ days, estimate the distance between the moon and the earth.

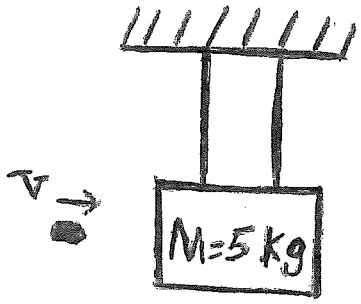
2- Two blocks A and B with respective masses m_A and m_B are connected via a massless string. Block B is on a frictionless table and block A is on a spring with spring constant k. Initially, the blocks are at rest and the spring is in its equilibrium position. Find the speed of block B when the spring is compressed by the amount $m_A g/k$. Determine the maximum compression of the spring and find the speed of block B when the spring is maximally compressed.



3– A train is moving at a constant speed of 30 km/h relative to the ground. People jump on the train in the direction of its motion with a speed of 10 km/h relative to the ground at a rate of one person per 30 seconds. If the average mass of each person is 65 kg, find the numerical value of average power required to keep the train moving.



4– A bullet of mass $m = 10$ g is fired into a wooden block of mass $M = 5$ kg hanging from the ceiling as shown in the figure. The bullet gets stuck in the block and the block+bullet system swings upward. The center of mass of this system rises a vertical distance 10 cm before stopping. What was the speed of the bullet before striking the block?



5- A solid uniform sphere of radius $R = 5$ cm is sliding on the floor with initial linear velocity of $v_0 = 5$ m/s and initial angular velocity $\omega_0 = 0$. The coefficient of kinetic friction between the sphere and the floor is $\mu_k = 0.15$. For how long does the sphere slide? Find the total distance that its center of mass travels while sliding.

6– A one-dimensional damped harmonic oscillator of mass m and natural frequency ω_0 is subject to a drag force $F_d = -2\beta m v$, where β is a constant and v is velocity. Write down the equation of motion for the dispalcement of the oscillator from its equilibrium position. Assuming that the period of damped oscillations is 1.001 times longer than that for undamped oscillations, find the damping factor β in terms of ω_0 and determine the decrease in the amplitude of oscillatons after 5 cycles.

7– A thin homogeneous plate lies in the $x - z$ plane. Its moment of inertia tensor in the xyz basis is given by

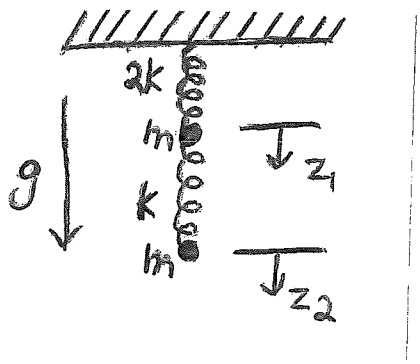
$$I = I_0 \begin{pmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 3 \end{pmatrix},$$

where I_0 is a constant. If the plate rotates about the x axis with a constant angular velocity ω, what torque must be applied to it to maintain its motion?

8– Two indetical masses m are attached to each other by a massless spring with spring constant k and suspended from the ceiling by a massless spring with spring constant $2k$ as shown in the figure. Determine the equilibrium positions of the two masses. Using coordinates z_1 and z_2 to describe the vertical displacements of the upper and lower masses from the equilibrium positions respectively, the equations of motion are given by

$$m\ddot{z}_1 + 3kz_1 = kz_2$$
$$m\ddot{z}_2 + kz_2 = kz_1 .$$

Find the normal modes and the corresponding frequencies for this system.



9- A high energy cosmic ray proton p can interact with a cosmic microwave background (CMB) photon γ and produce a neutral pion π_0 via the following reaction

$$p + \gamma \rightarrow p + \pi_0 .$$

Considering a head-on collision as shown in the figure, and assuming that the incident proton is in the ultra-relativistic regime ($E_p \gg m_p c^2$), determine the minimum (threshold) proton energy for this process to occur. Use $m_p c^2 = 940$ MeV, $E_\gamma = 300$ μeV (1 μeV=10^{-6} eV), and $m_{\pi_0} c^2 = 135$ MeV. (Hint: Go to the center of mass frame where the total momentum of the system is zero.)

10– A bead of mass m slides without a friction on a wire that is bent in the shape of a curve whose equation in cylndrical coordinates (ρ, ϕ, z) is given by $z = \rho^4$. The wire is spun with constant angular velocity ω about its axis of symmetry as shown in the figure. Write down a Lagrangian for this system in terms of ρ as the general coordinate and derive the equation of motion of the bead. Find the positions of equilibrium and determine whether they are stable or unstable.