Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico

Fall Semester 2015

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- **no notes! but calculator yes**
- *total time is 3 hours*

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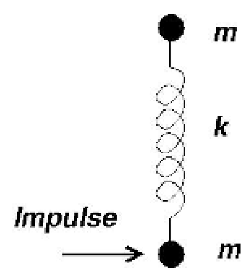
1. Starting with Kepler's 2^{nd} law and Newton's law of gravity, derive the orbital period T as a function of the orbital radius r, the mass of the sun and Newton's G. Assume orbits are circular. Show that this is a simplified form of Kepler's 3^{nd} law.

2. Consider an attractive central force $F(r) = -k/r^3$ where r is the distance to the origin. Obtain the equation for the orbit. Show that there is no stable equilibrium for any value of ℓ.

3. A system consisting of two pucks of equal mass m and connected by a massless spring (with spring constant k and un-stretched length r_0) is initially at rest on a horizontal, frictionless table with the spring at it's uncompressed length. One mass is then given an essentially instantaneous impulse I perpendicular to the direction connecting the masses. Find the effective potential and use it to show that in the subsequent motion r is never less than r_0 (*i.e.* r_0 is one of the turning points of the motion).



4. Consider a one dimensional simple harmonic oscillator with angular frequency ω and amplitude A. The oscillator spends more time near it's turning points at $x = \pm A$ then at its equilibrium point at $x = 0$. Show this by obtaining an expression for the fraction of a complete period that the oscillator spends within a small interval between x and $x + \Delta x$. Prove that this time interval is finite in the neighborhood of the turning point.

5. Find the height above the center of the billiard ball at which the ball should be struck so that it will roll with no initial slipping. Assume that the cue delivers a horizontal impulse to the ball.

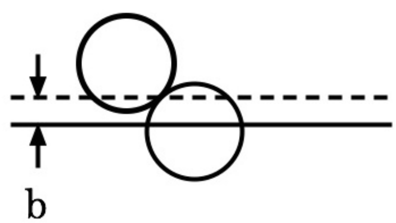
6. The average rate at which solar radiant energy reaches Earth is approximately 1.4 kW/m^2. Assume that all this energy results from the conversion of mass to energy. Calculate the rate at which the solar mass is being lost, and use this result to estimate of the lifetime of the sun, in years. (The earth's orbital radius is 1.5×10^{11} m and the solar mass 2×10^{30} kg)

7. Consider the electron capture reaction of an electron by argon, where both argon atom and electron are assumed to be at rest, which yields a chlorine atom and a neutrino:

$$e^- + {}^{37}_{18}Ar \rightarrow {}^{37}_{17}Cl + \nu_e$$

The atomic masses (which include the bound electrons) are: ${}^{37}_{18}Ar = 36.966776$ u, ${}^{37}_{17}Cl = 36.965903$ u, the electron mass is 5.485799×10^{-4} u and you can take $m_{\nu_e}c^2 = 0$ eV. The atomic mass unit $u = 931.5$ MeV/c^2. What are the final kinetic energies of the neutrino (ν_e) and the chlorine atom in eV?

8. Consider a two dimensional elastic scattering problem that would be applied to colliding pucks (radius R) on an ice rink (*i.e.* frictionless horizontal plane). Further assume that the surfaces of the pucks are frictionless. Show that the relation between the impact parameter b and the scattering angle θ is $b/R = \cos(\theta/2)$ where the impact parameter is defined as the distance between a line throuth the center of one puck in the direction of the relative velocity and a parallel line through the point of contact as in the figure below. Derive the differential cross section $d\sigma/d\theta$ in terms of the impact parameter R and the scattering angle θ. Show that the total cross section is $2R$.



9. Consider a very long string under tension τ that has a mass density discontinuity at the middle of the string (μ_1 for $x < 0$ and μ_2 for $x > 0$), Define the ratio of wave numbers as $r = k_2/k_1 = \sqrt{\mu_2/\mu_1}$. Find the value of r for which the transmission coefficient T is a maximum (r_{max}) and the value of T at r_{max}. Recall that the transmission coefficient T and reflection coefficient R satisfy $R + T = 1$. Sketch the transmission coefficient $T(r)$ being careful to show r_{max} and the qualitative behavior at $r = 0$ and $r \rightarrow \infty$.

10. A lawn sprinkler is made from a half-spherical cap (max angle $\theta = 45°$, radius a) with a large number of identical holes, with density $n(\theta)$. Determine $n(\theta)$ such that the water is uniformly sprinkled over a circular area. The bottom of the half-spherical cap is level with the lawn. Assume that the size of the cap is negligible compared to the size of the lawn to be watered and neglect air resistance.