Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico
Fall Semester 2014

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- **no notes!**
- *total time is 3 hours*

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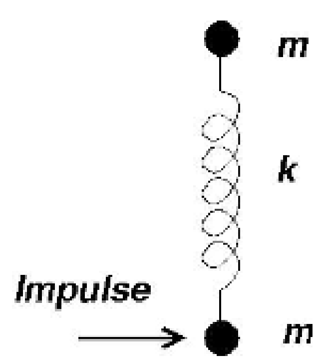
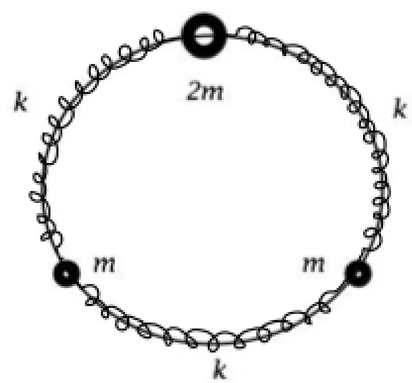
1. A uniform rod of length ℓ and mass m stands vertically upright on a smooth floor and then tips over. Assuming the surface is frictionless, what is the rod's angular velocity when it hits the floor?

2. A system consisting of two pucks of equal mass m and connected by a massless spring (with spring constant k) is initially at rest on a horizontal, frictionless table with the spring at it's uncompressed length. One mass is then given an essentially instantaneous impulse J perpendicular to the direction connecting the masses. Write an equation for the minimum and maximum length of the spring in the subsequent motion. You need not solve this equation for the maximum length of the spring, but what is the minimum length? Make a qualitative sketch showing a graphical solution for the maximum length of the spring.

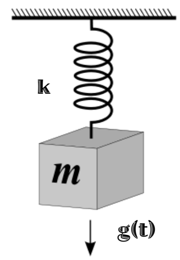


3. A rocket (starting from rest) moves in free space and ejects exhaust with constant relative veolcity v_{ex}. Find the velocity of the rocket as a function of mass m, given its intial mass m_0. How does this result depend on the rate dm/dt at which the exhaust is ejected?

4. Consider a frictionless rigid horizontal hoop of radius R. Onto this hoop are threaded three beads with masses $2m$, m, and m. Each pair of beads is connected by three identical springs, each with force constant k. Show that the normal frequencies are $0, \sqrt{2}\omega_0, \sqrt{3}\omega_0$. Explain the physical significance of the zero frequency mode.



5. The New Mexico Railrunner^{TM} commuter train is equipped with an accelerometer to measure the smoothness of the ride. The device consists of a mass m hanging from a spring with stiffness constant k: When the train is at rest, the mass experiences the constant acceleration of gravity g. When the train moves along the tracks, however, the acceleration $g(t) = g + g' \sin \Omega t$ changes sinusoidally in time with a frequency Ω and an amplitude g'. After the train has been moving for some time, the mass assumes a steady state oscillation at the frequency Ω. If $\Omega \neq \sqrt{k/m}$ what is the displacement x of the mass as a function of time t in steady state?

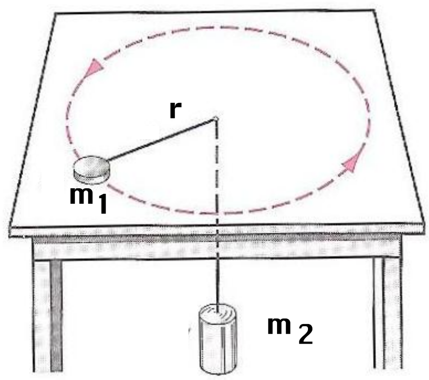


6. A small mass m_1 slides without friction on a level surface. It is connected by a string through a tiny hole in the plane to a second mass m_2 that is hanging below the plane. Initially, the first mass is given an an angular velocity ω_0 so that it moves in a circular trajectory of radius r_0 and angular momentum $\ell = m_1 r_0^2 \omega_0$. If the second mass is now given a small displacement in the vertical direction, the system oscillates with a frequency . Starting with the radial equation of motion,

$$(m_1 + m_2)\ddot{r} = \frac{\ell^2}{m_1 r^3} - m_2 g$$

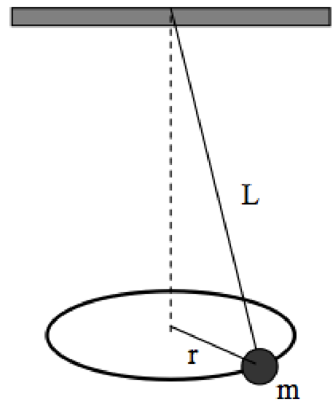
show that

$$\Omega = \omega_0 \sqrt{\frac{3m_1}{m_1 + m_2}}$$

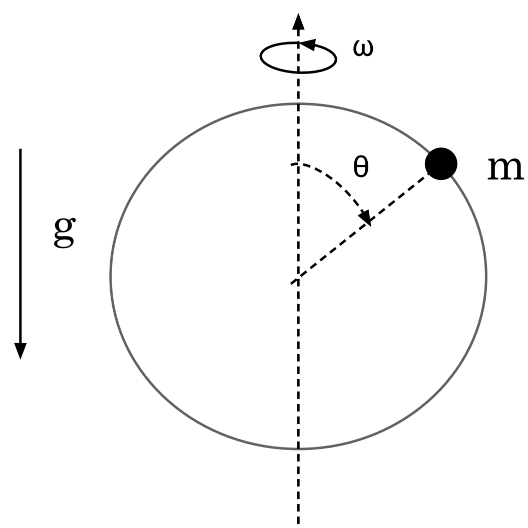


7. Evidence for dark matter comes from "flat" rotation curves of galaxies. Assume that the observed matter moves in circular orbits about the center of the galaxy and that the velocity of the matter as a function of the radius $v(r)$ is a constant. Also assume the motion of the observed matter is purely due to the gravity of the dark matter (mass of luminous matter is negligible) and the dark matter is distributed with spherical symmetry about the center of the galaxy. What is the density $\rho(r)$ of the dark matter as a function of radius?

8. In the conical pendulum below, the bob has a mass m, the massless string has a length L, and the bob follows a circular path of radius r. What are a) the tension in the string τ, and b) the period of motion T?



9. A bead of mass m slides freely on a vertical hoop that of radius R that is rotated with constant angular velocity ω about the vertical direction (see figure). Write a Lagrangian for the system in terms of the generalized coordinate θ.



10. The principle of the synchrotron proton accelerator is to keep the beam radius constant by increasing the strength of the magnetic field with increasing particle momentum. In order to accomplish this, the beam frequency must be known as a function of proton momentum. Derive this equation. Neglect proton synchrotron radiation. Check that in the limit of $p >> mc$ the frequency becomes independent of the proton momentum.