Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico
Fall Semester 2013

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours*

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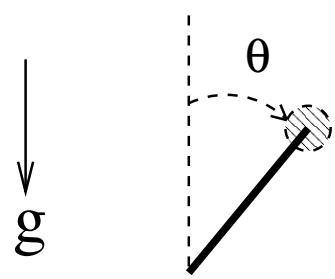
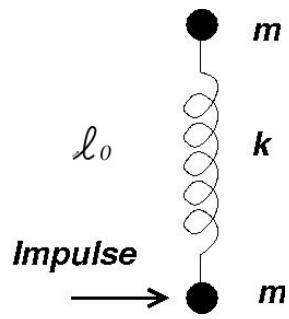
1. Consider a system of two identical masses (m) connected by a spring (constant k) which is constrained to move along the direction connecting the two masses, but can move freely along that direction. At $t = 0$ mass 1 is at rest and mass two has a velocity $\vec{v} = v_2(0)\hat{x}$ and the spring is at its un-stretched length (ℓ). What is the velocity of mass 1 as a function of time t?

2. A particle of mass m, attached to a rigid rod of length ℓ, rotates in a vertical plane with constant angular speed ω. What is the magnitude of the force that must be applied to the mass by the rod as a function of angle θ? Take $\theta = 0$ to be the vertical direction.

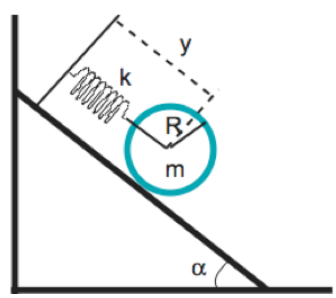


3. Suppose that the earth was an infinite flat slab of thickness t with uniform density ρ_0. Calculate t in order that this flat earth has the same acceleration due to gravity at the surface ($= g$) as the round earth.

4. A system consisting of two pucks of equal mass m and connected by a massless spring (with spring constant k) is initially at rest on a horizontal, frictionless table with the spring at its uncompressed length ℓ_0. One mass is then given an essentially instantaneous impulse I perpendicular to the direction connecting the masses. Write an equation that whose solution gives the minimum and maximum length of the spring in the subsequent motion. What is the minimum length of the spring? Make a qualitative sketch showing a graphical solution for the maximum length of the spring.



5. A hoop of mass, m, and radius R rolls without slipping down a plane as shown in the figure below. A massless spring is attached to a massless axle. Write a Lagrangian for the system and use it to derive the equation of motion in the y coordinate.

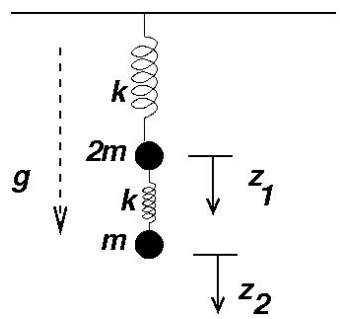


6. Two masses $2m$ and m are attached to each other by a massless spring with spring constant k and and suspended from the ceiling by an identical spring (refer to figure). Only vertical displacements are considered. Using coordinates z_1 and z_2 to describe the displacements from equilibrium of the upper and lower masses respectively, the equations of motion are given by,

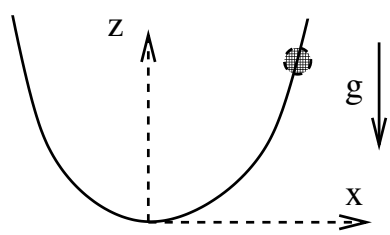
$$2m\ddot{z}_1 + 2kz_1 = kz_2$$

$$m\ddot{z}_2 + kz_2 = kz_1$$

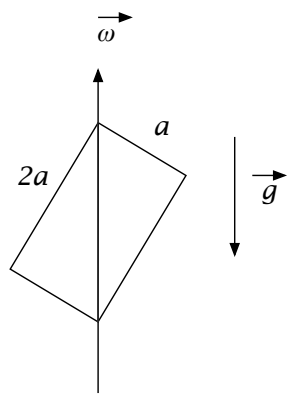
Determine the normal modes and normal mode frequencies of this system.



7. A particle of mass m slides without friction on a bent wire subject to the force of gravity. Taking the vertical direction to be $\hat{z}$ and the horizontal direction to be $\hat{x}$, the shape of the wire is given by $z = ax^2/2$. Write the Lagrangian taking x as the generalized coordinate.



8. A door of mass m (uniform mass per area), width a and height 2a is rotated about a vertical axle through two diagonal corners. The bearings supporting the plate are mounted just at the corners. If it is then rotated at constant angular frequency ω, find the torque that must be supplied by the bearings.



9. A particle of mass m is bound to the origin by a force $\vec{F} = -k\vec{r}$. The Lagrangian of the system may therefore be taken to be,

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{1}{2}kr^2.$$

Show that circular orbits are stable.

10. A particle called a neutral pion π_0 decays to two photons. Suppose the π_0 has some initial momentum in the lab in the x direction and one photon is detected in the $+x$ direction with energy E_+ and the other in the $-x$ direction with energy E_-. From these observations, what is the speed of the pion in the lab?