Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours*

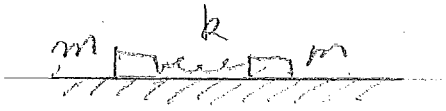
---

moments of inertia tensors for selected rigid bodies:

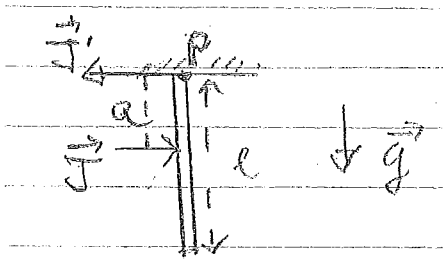
thin rod about axis through center and perpendicular to length ℓ	$\frac{1}{12}m\ell^2$
cylinder of about central axis of length ℓ	$\frac{1}{2}m\ell^2$
sphere of radius ℓ	$\frac{2}{5}m\ell^2$
slab (width a length b) about perpendicular axis through center	$\frac{1}{12}m(a^2+b^2)$

---

1. Two identical carts (of mass m) are free to move on a frictionless, straight horizontal track. The masses are connected by a spring of constant k and un-stretched length l_0. Initially the masses are a distance l_0 apart with the mass on the left having a speed v_0 to the right and the mass on the right at rest. Find the position of mass on the left as a function of time.

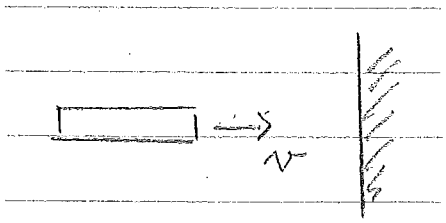


2. A stick of length ℓ is suspended by one end at point P so that it hangs vertically and so that the top end of the stick does not move. A horizontal impulse J is applied perpendicular to the stick a distance a below the point of suspension. In general there will be an opposite impulse J' that must be given at the top of the stick (point P) to keep its point of suspension fixed. Find the distance a such that $J' = 0$.

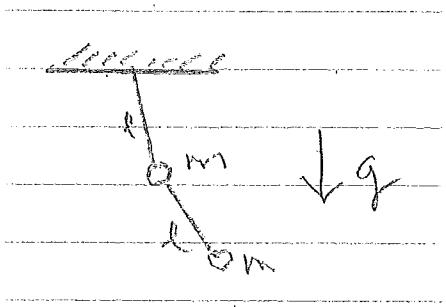


3. Ignoring air friction, what is the minimum initial speed needed for a space ship of mass m_s to get from the earth to the moon? (Take the earth-moon separation to be R and the masses and radii to be r_e, m_e, r_m, m_m.)

4. A relativistic meter stick moves with speed v in the lab. It collides head-on with an impenetrable wall completely inelastically, thereby coming to rest in the lab frame. What is the maximum length of the stick in the lab after the collision?



5. Write a Lagrangian for the coplanar double oscillator assuming massless connecting rods using <u>two</u> (clearly defined) generalized coordinates (see figure). Don't bother simplifying the kinetic energy term.

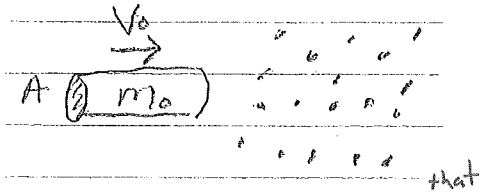


6. A long string of linear density λ is under tension T. A point mass m is attached at a particular point on the string. A wave of angular frequency ω is incident from the left. What fraction of the incident energy is reflected by the point mass? (Hint– the quantum-mechanical analog might be helpful.)
7. A thin homogeneous plate lies in the $x-y$ plane. Its moment of inertia tensor in the x, y, z basis is given by

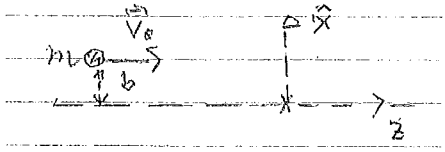
$$I = \sigma\ell^4 \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

where σ is the mass per unit area and ℓ is a length. If the plate is rotated about the $\hat{x}$-axis with constant angular frequency ω, what torque must be applied to the plate to keep the rotation axis pointing in the x direction?

8. A spacecraft of mass m_0 and cross-sectional area A is coasting with a velocity v_0 when it encounters a stationary dust cloud of density ρ. Assuming that all the dust sticks to the spacecraft and that A remains unchanged, find the subsequent motion, i.e. $v(t)$ of the space craft.



9. A particle of mass m collides with a fixed force center acts on the mass with a repulsive force of k/r^2. It has initial velocity v_0 in the z direction and impact parameter b. Find the distance of closest approach of the mass.



10. A large, spherically symmetric collection of point particles of mass m move in circular orbits about a common center each with the same kinetic energy. If the only force acting is the mutual gravitational attraction of the particles, find the particle density (in the continuum limit) as a function of r from the center in order that the density remain constant in time.