Department of Physics and Astronomy, University of New Mexico

Classical Mechanics Preliminary Examination

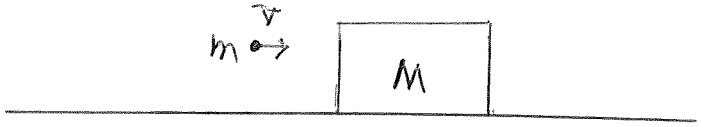
Fall 2010

Instructions:

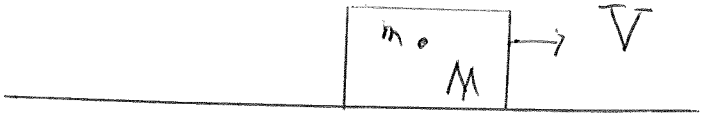
- The exam consists of 10 short-answer problems (10 points each).
- Partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed.
- Total time: 3 hours.

1– A bullet with mass m and speed v hits a wooden block of mass M that is situated at rest on a frictionless surface. It penetrates the block and gets trapped inside it as a result of a constant retardation force F_{ret} that opposes relative motion between the two objects. Find the common speed of the bullet and the block V, and the penetration length l in terms of m, M, v and F_{ret}.

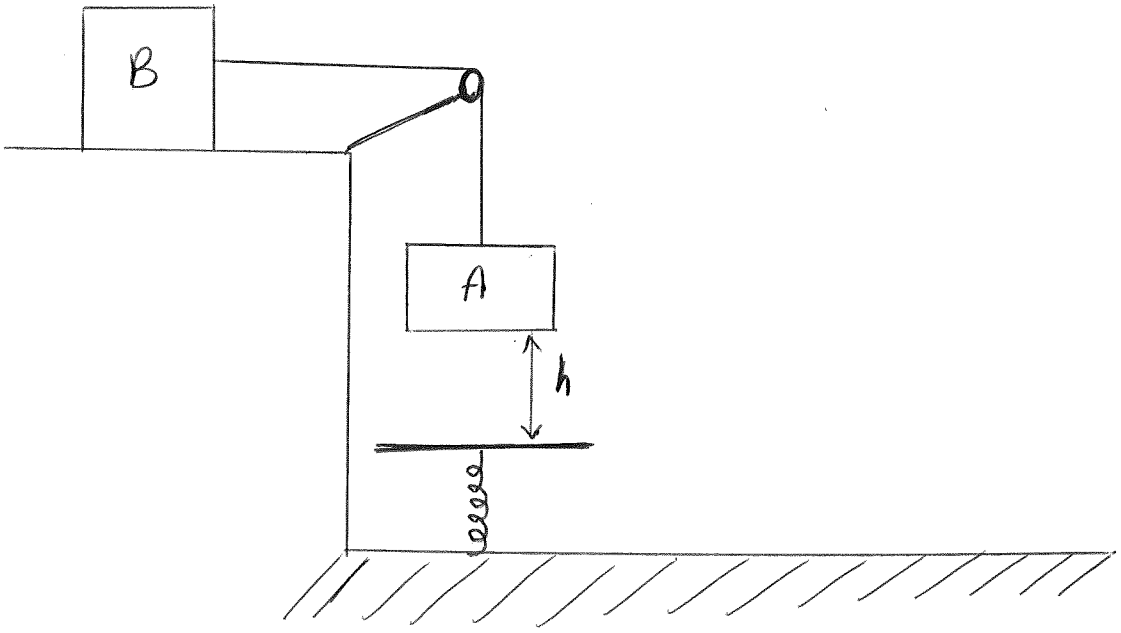
Before



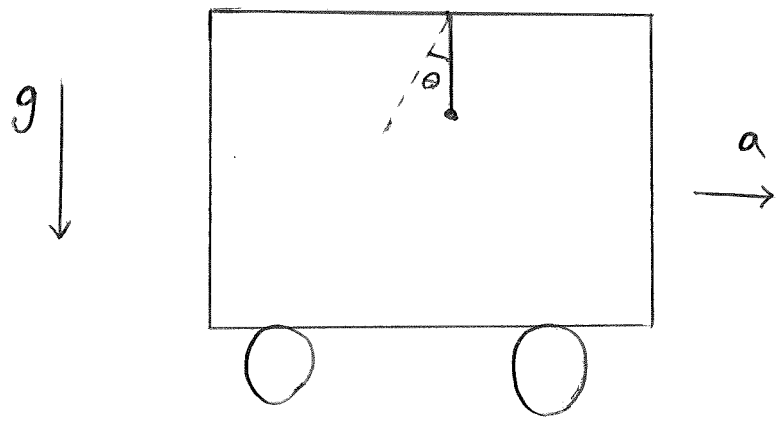
After



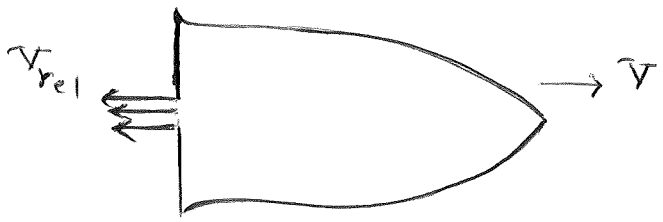
2– Two blocks A and B with respective masses m_A and m_B are connected via a string. Block B is on a frictionless table, and block A is hanging at a vertical distance h from a spring with spring constant k that is at its equilibrium position. The blocks are initially at rest. Find the velocity of A and B when the spring is compressed by an amount $\delta y = m_A g/k$. Determine the maximum compression $\delta y_{\max}$ of the spring in terms of m_A, m_B, g and k. (Hint: what happens to the motion of blocks when $\delta y = m_A g/k$?)



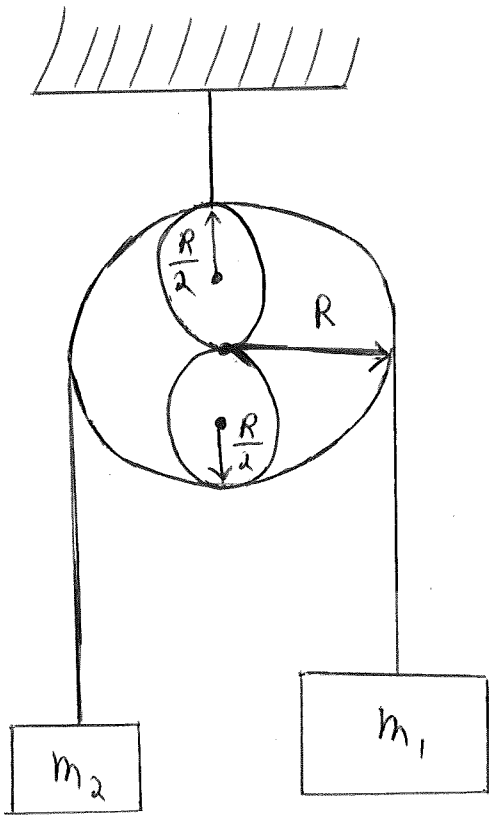
3– A simple pendulum is suspended vertically inside a wagon. The wagon starts moving with a constant acceleration a. Assuming $a \ll g$, find the frequency of oscillations of the pendulum and its amplitude with respect to an observer in the wagon.



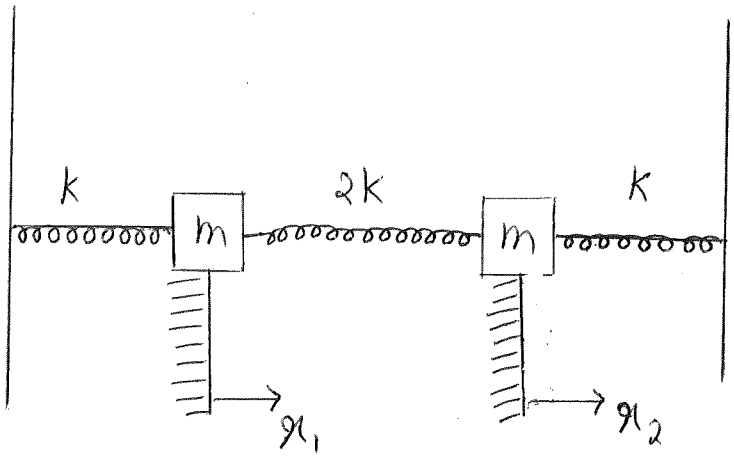
4– A rocket ejects pressurized air with constat relative velocity v_{rel} and moves horizontally. Starting from the rest and an initial mass m_1, find the speed of the rocket when its mass is m_2 ($m_2 < m_1$). How does this result depend on the rate $r = dm/dt$ at which the air is ejected?



5– A system consists of a pulley and two masses m_1, m_2 hanging from it as shown in the figure. The pulley is a disk of radius R with constant mass density σ that has two circular holes with radius $R/2$. Find the angular acceleration of the pulley. (The moment of inertia of a uniform disk of mass M and radius R about its center is $I = MR^2/2$).



6– Two blocks of equal mass m are connected by three springs with spring constant k, $2k$, k to each other and two walls as shown in the figure. Find and describe each of the normal frequencies of oscillation for this system.



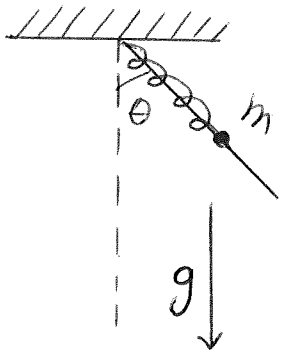
7– A particle of mass m moving in three dimensions is attracted to the origin by the gravitational force of a much heavier object. It can be shown that the radial motion is governed by the following equation

$$m\ddot{r} = -\frac{k}{r^2} + \frac{l^2}{mr^3}\,,$$

where k is a constant and l is the angular momentum. Determine an equilibrium radius r_0 in terms of k, l and m. If the particle is put near that equilibrium radius, $r = r_0 + \epsilon$ (where $\epsilon \ll r_0$), it will have an oscillatory radial motion about r_0. What will be the frequency of that oscillation?

8– By using conservation laws, prove that emission of a photon by a free electron (i.e., $e \rightarrow e + \gamma$) cannot occur.

9– A small bead of mass m is attached to a spring with spring constant k and can slide without friction on a massless rod as shown in the figure. The spring has zero length at its equilibrium position. The rod hangs from one of its endpoints and is free to rotate about it in the plane of the page. Write the Lagrangian for this system and derive the equations of motion for the bead in a suitable coordinate system.



10– Consider a bead of mass m constrained to slide without friction along a rigid wire that rotates about the vertical at a fixed angle θ with constant angular velocity ω. Write down the Lagrangian in terms of z as the general coordinate. Find the equation of motion of the bead, and determine whether there are positions of equilibrium. If there are equilibrium positions, are they stable?