Preliminary Examination: Mechanics

Department of Physics and Astronomy
University of New Mexico
Fall 2007

Instructions:

- *the exam consists of 10 problems, 10 points each;*
- *partial credit will be given if merited;*
- *personal notes on two sides of* 8×11 *page are allowed;*
- *total time is 3 hours.*

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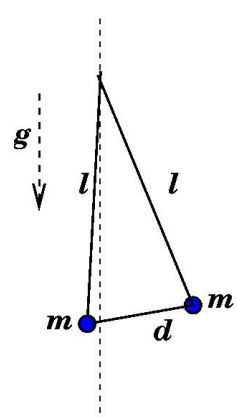
1. A rock of mass m_0 sits at the bottom of a super-deap mine shaft that is 1/3 the radius of the earth. Ignoring air friction, what is the minimum speed with which the rock can be launched to reach the surface of the earth? Express your answer as a ratio to the earth's escape velocity.

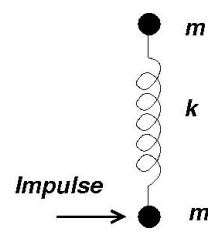
2. Two spheres are of the same mass and the same diameter but one is solid and the other is hollow. Describe a non-destructive experiment to determine which is which. Be sure to clearly state the experimental conclusion, for example "thus this is the solid sphere".

3. A physical pendulum is made of rigid rods of negligible mass in the shape of an isosceles triangle with two equal masses attached as shown. Consider only motions in the plane of the triangle. Find the frequency of small oscillations of the pendulum.



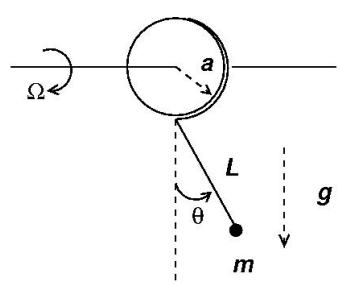
4. A uniform rod of length ℓ is attached to the floor by a fixed pivot point. It stands vertically and then tips over. What is the rod's angular velocity when it hits the floor?

5. A system consisting of two pucks of equal mass m and connected by a massless spring (with spring constant k) is initially at rest on a horizontal, frictionless table with the spring at it's uncompressed length. One mass is then given an essentially instantaneous impulse I perpendicular to the direction connecting the masses. Write an equation that whose solution gives the minimum and maximum length of the spring it the subsequent motion. What is the minimum length of the spring? Make a qualitative sketch showing a graphical solution for the maximum length of the spring.



6. A neutron of kinetic energy 2 MeV collides head-on in an elastic collision with a nuclei of mass number A which is initially at rest. What is the smallest energy the neutron can have following the collision?

7. A simple pendulum is suspended from the rim of a wheel which rotates within the vertical plane with constant angular velocity Ω. (refer to figure) Consider only motion in which the pendulum bob swings in the plane of the wheel. Ignore any potential tangling of the string with the wheel or axle. Write the exact equation of motion for the bob in terms of the angle θ.



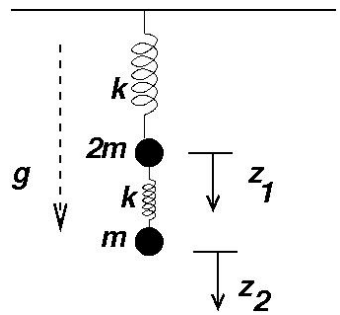
8. Consider a one dimensional simple harmonic oscillator with angular frequency ω and amplitude A. The oscillator spends more time near it's turning points at $x = \pm A$ then at its equilibrium point at $x = 0$. Show this by obtaining an expression for the fraction of a complete period that the oscillator spends within a small interval Δx at position x.

9. Two masses $2m$ and m are attached to each other by a massless spring with spring constant k and and suspended from the ceiling by an identical spring (refer to figure). Only vertical displacements are considered. Using coordinates z_1 and z_2 to describe the displacements from equilibrium of the upper and lower masses respectively, the equations of motion are given by,

$$2m\ddot{z}_1 + 2kz_1 = kz_2$$

$$m\ddot{z}_2 + kz_2 = kz_1$$

Determine the normal modes and normal mode frequencies of this system.



10. Ultra-high energy protons propagating through the universe interact with the cosmic-microwave background photons. What is the proton energy for the resonance process

$$p + \gamma \rightarrow N^\star \rightarrow p\pi^0 \text{ or } n\pi^+$$

where $m_{N^\star} = 1240$ MeV, $m_p = 940$ MeV and the mean CMB photon energy is 10^{-4} eV?