Preliminary Examination: Classical Mechanics

Department of Physics and Astronomy

University of New Mexico

Fall 2004

Instructions:

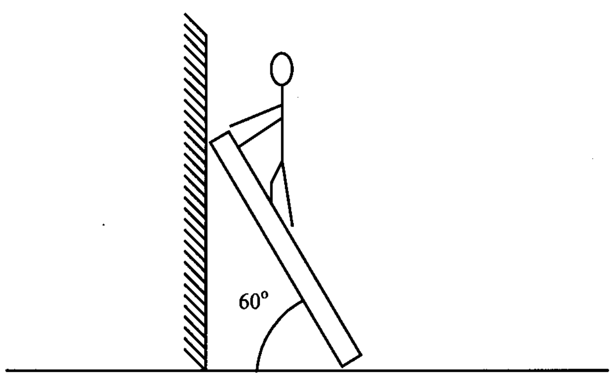
- The exam consists of two parts: 5 short answers (6 points each) and your choice of 2 out 3 long answer problems (35 points each).
- Where possible, show all work, partial credit will be given.
- Personal notes on two sides of a 8X11 page are allowed.
- Total time: 3 hours

Good luck!

Short answers:

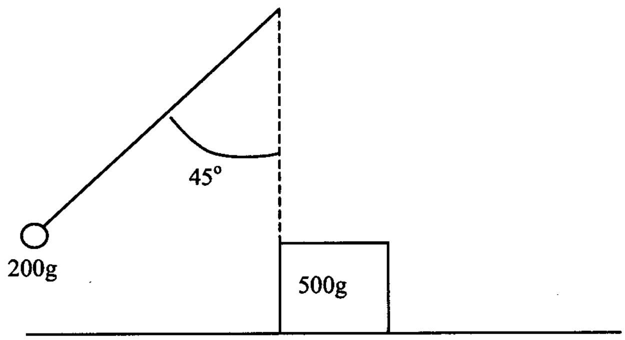
S1. A spring gun shoots a dart of mass 100g by compressing a spring with constant k = 80N/m a distance of 0.1m. When the dart leaves the gun, it lodges in a wooden door 2m high and 1m wide weighing 20kg on frictionless hinges. If the shooter hits the door 0.5m from the hinges and the dart strikes the door normal to its plane, what is the angular momentum of the door about the hinges after the hit?

S2. A 100kg man is standing on a 3m long (weightless) ladder and his mass is centered 2/3 up the ladder. The ladder is leaning ($\theta = 60^\circ$) against a frictionless wall. The floor is slippery, with a coefficient of static friction of 0.2. Determine if this situation is stable.



S3. A planet ($m = 10^{10}$ kg) is in an elliptical orbit around the sun ($M = 2\times 10^{30}\,kg$). The distance between the planet and the sun at the perihelion (point of closest approach) is $10^{13}\,m$, and at that point the total energy (kinetic plus potential) is $E = -1.4\times 10^{16}$ J. What is the magnitude of the velocity of the planet at the aphelion (furthest point) from the sun? ($G = 6.7\text{x}10^{-11}\ \text{Nm}^2/\text{kg}^2$)

S4. A 200g steel ball hangs on a 1m long string. The ball is pulled sideways so that the string is at a 45° angle, then released. At the very bottom of its swing, it strikes a 500g steel block that is resting on a frictionless table, colliding elastically. To what angle does the ball rebound?

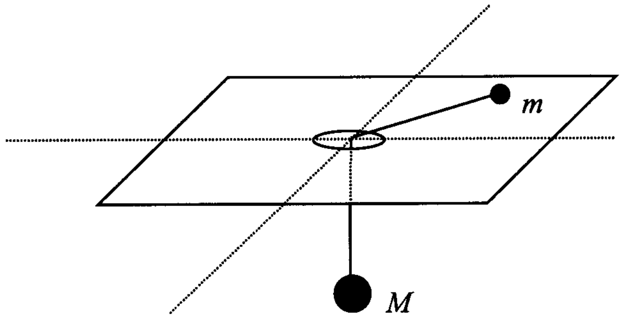


S5. Two ideal pendulums of length l and with mass m are connected with a massless spring, with spring constant κ. What are the normal modes and frequencies of oscillation? At time t=0, one pendulum is slightly displaced from equilibrium and the other is held at rest. After a time τ, the first pendulum will be at rest and the other swinging. Find τ.

Long Answers: Pick two out of three problems below

L1. A mass m moves on a frictionless table. It is tied to a string which runs through a hole in the table. A mass M hangs from the other end of the string and acted upon by gravity. M is constrained to move vertically.

(a) Choose appropriate coordinates and write a Lagrangian for the system.
(b) Describe the symmetries and conserved quantities.
(c) Find a class of solutions in which the mass M is stationary



L2. A bead of mass $m = 1$ kg slides on a frictionless wire. The wire is held rigidly in the shape of a circular hoop of radius $R = 1$ m, as shown in the figure. Initially the bead is at rest at the top of the hoop. The bead will begin to slide through an angle θ when slightly perturbed from this unstable position. The moment of inertia of the bead may be neglected.

(a) Show that, for small angles, θ evolves according to the equation of motion

$$\ddot{\theta} - \lambda^2\theta \approx 0, \text{ where } \lambda = g/R.$$

(b) Suppose that the bead is balanced at the top of the hoop, $\theta(0) = 0$, and given a very light push such that initial angular velocity is $\dot{\theta}(0) = 10^{-17}s^{-1}$, is on the order of the minimum allowed by the uncertainty principle. Solve the equation of motion and determine how long (in seconds) it will take the bead to slide through an angle of 30 degrees.

(c) Using the fact that energy is a conserved quantity, show that the time Δt it takes a bead to slide between any two angles θ_1 and θ_2 is given *exactly* by

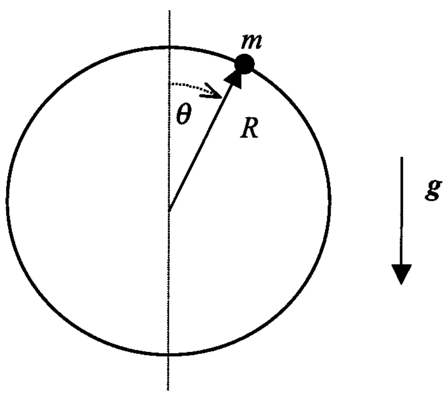
$$\Delta t = \int_{\theta_1}^{\theta_2} \frac{d\theta}{\sqrt{\frac{2E}{mR^2} - \frac{2g}{R}\cos\theta}}.$$

where E is the total energy and the potential energy has been taken to be zero at θ= 90 degrees.

(d) After reaching the angle of 30 degrees, approximately how much longer (in seconds) will it take the bead to slide the rest of the way to the bottom of the hoop?

Useful integral:

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\sqrt{1-\cos\theta}} = \sqrt{2}\ln\left[\frac{\sin(\theta_1/2)}{\sin(\theta_2/2)}\left(\frac{1-\cos(\theta_2/2)}{1-\cos(\theta_1/2)}\right)\right].$$



L3. A bowling ball is released from rest at the top of an inclined surface. The surface consists of a straight, board, 3 meters in length, which is inclined at an angle of 60 degrees with respect to the horizontal. The coefficient of static friction between the board and the ball is 0.60, and the coefficient of sliding (kinetic) friction is 0.40. The radius of the bowling ball is 10 cm.

(a) Does the bowling ball roll without slipping? How much time (in seconds) does it take for the ball to reach the end of the incline?

(b) Suppose that the ball is given a back-spin when it is released from rest, so that its initial angular velocity is 50.0 rad/s. (See the figure below.) How much time (in seconds) does it take for the ball to reach the end of the incline in this case?