Preliminary Examination: Astronomy

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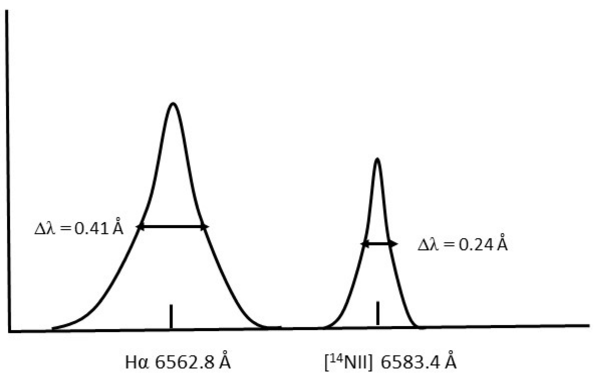
Instructions:

- Answer 8 of the 10 questions (10 points each)
- Total time for the test is three hours
- Partial credit will be given if merited, so show your work towards completion of the problems
- No notes allowed - potentially useful constants and equations are provided below
- Calculators (provided) allowed

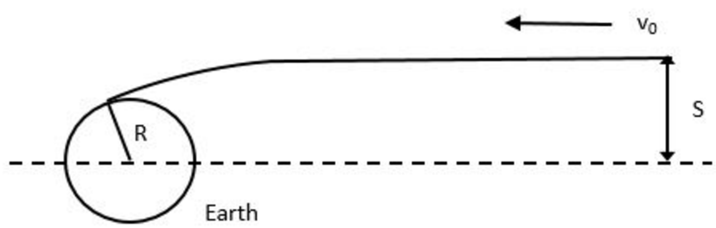
Speed of light	$c = 3 \times 10^8$ m s^{-1}
Planck's constant	$h = 6.626 \times 10^{-34}$ J s$= 4.136 \times 10^{-15}$ eV s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Stefan-Boltzmann's constant	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J K^{-1} $= 8.62 \times 10^{-5}$ eV K^{-1}
Mass of the Sun	1 M$_\odot$ $= 1.99 \times 10^{30}$ kg
Luminosity of the Sun	1 L$_\odot$ $= 3.8 \times 10^{26}$ W
Radius of the Sun	1 R$_\odot$ $= 6.96 \times 10^8$ m
Mass of the Earth	1 M$_E$ $= 5.97 \times 10^{24}$ kg
Radius of the Earth	1 R$_E$ $= 6.37 \times 10^6$ m
Mass of a hydrogen atom	1 m$_H$ $= 1.67 \times 10^{-27}$ kg
Astronomical unit	1 AU $= 1.496 \times 10^{11}$ m
Parsec	1 pc = 3.26 ly $= 3.086 \times 10^{16}$ m = 206,265 AU
1 eV	1.6×10^{-19} J
1 Jy	10^{-26} W m^{-2} Hz^{-1}

$I = I_0 e^{-\tau}$	Radiative transfer (absorption only, $\tau = n\sigma s$)
$\lambda_{max} = 0.0029/T$ m K	Wien's Law
$F = \sigma T^4$	Stefan-Boltzmann Law
$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$	Planck Function
P=nkT	Ideal gas law, where $n = \rho/\mu m_H$
KE $= \frac{3}{2}kT$ (per particle)	with the average velocity per particle $v = (3kT/m)^{1/2}$
$P_{rad} = aT^4/3$	Radiation pressure
$v = c(\lambda_{obs} - \lambda_0)/\lambda_0$	Doppler velocity
$\nu_\infty/\nu_0 = (1 - 2GM/r_0c^2)^{1/2}$	Gravitational redshift
$R_{sch} = 2GM/c^2$	Schwarzschild radius
$R_S = \left(\frac{3N}{4\pi\alpha}\right)^{1/3} n_e^{-2/3}$ cm	Strömgren radius
$R(t) = (1+z)^{-1}$	Scale factor
$T_0 = R(t)T(t)$	Scale factor effect on temperature
$R(t) = (t/t_0)^{1/2}$	Scale factor for a radiation dominated Universe
$R(t) = (t/t_0)^{2/3}$	Scale factor for a matter dominated Universe
$R(t) = e^{H_0(t-t_0)}$	Scale factor for a cosmological constant dominated Universe

1. Below is a spectrum of the Hα and [^{14}NII] lines in an HII region at rest. The indicated widths of the lines are due to Doppler broadening in the gas $\left(\frac{\Delta\lambda}{\lambda} = \frac{v}{c}\right)$. Take the velocity corresponding to this width as a measure of the combined thermal velocity $\left(\sqrt{\frac{3kT}{m}}\right)$ and turbulent velocity added in quadrature. Find the temperature and turbulent velocity of the gas.



2. A meteor approaches the Earth with a speed v_0 when it is at a very large distance from the Earth. Show, considering conservation laws, that the meteor will strike the Earth, at least at gracing incidence, if its impact parameter S is given by $S \leq [R^2 + 2GMRv_o^{-2}]^{1/2}$



3. a. Describe what is meant by a system being in *virial equilibrium.*

b. NGC1316 is a giant elliptical galaxy with an approximate mass of 2×10^{12} $M_\odot$ within a radius of 400 kpc. How long would it take for a star at the edge to orbit the center once?

c. Assume that NGC1316 has been capturing smaller satellite galaxies up til present time, and given your answer in b), would you expect the outer parts of NGC1316 to be in virial equilibrium? Why or why not?

4. A pure hydrogen gas cloud of number density $n = 10$ cm^{-3} surrounds an O5V star that generates a Lyman continuum flux of $N = 3\times10^{49}$ s^{-1}. The rate at which hydrogen recombines is a function of temperature but can be taken to be $\alpha = 3 \times 10^{-13}$ cm^3 s^{-1}.

   a. What is the physical meaning of the Lyman continuum flux, N?

   b. Calculate the size of the Strömgren sphere associated with this star. Express your answer in parsecs.

   c. What do the letters and numbers in the designation of stellar type (e.g., 'O5V') signify? Explain all three components.

5. Quasars can be useful for studying gas between us and the quasar. This can include both gas clouds inside galaxies, and gas clouds in the intergalactic medium. Assume you observe carbon absorption lines in the spectrum of a background quasar. From the relative strengths of the lines, the temperature of the cloud is estimated to be 20 K.

   a. If the redshift of the cloud is $z = 1.776$, how does the temperature of the cloud compare to the temperature of the CMB at the same redshift?

   b. Based on the above, do you think the absorption line arise in the intergalactic medium, or inside a galaxy? Motivate your answer.

6. The Friedmann equation describes the expansion of the universe, and one of the forms we are used to see this equation includes matter, relativistic particles and dark energy density as:

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G(\rho_m + \rho_{rel} + \rho_\Lambda)\right] R^2 = -kc^2$$

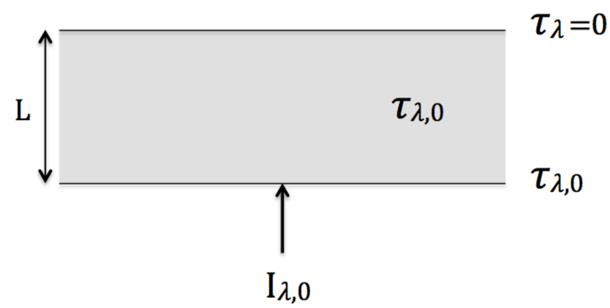
   a. Show that, in a radiation-dominated universe, the curvature $|1 - \Omega|$ is proportional to R^2, where R is the expansion parameter. *Hint: Recall the definition of the critical density,* $\rho_c = \frac{3H^2}{8\pi G}$.

   b. Explain how this leads to the "flatness problem" in the standard Big Bang model.

7. The radiative transfer equation describes the spectral energy distribution of radiation passing through matter, with optical depth τ_λ and source function S_λ:

$$I_\lambda(0) = I_{\lambda,0}e^{-\tau_{\lambda,0}} - \int_{\tau_{\lambda,0}}^{0} S_\lambda e^{-\tau_\lambda} d\tau_\lambda$$

where $I_\lambda(0)$ is the emergent flux. Consider a horizontal plane-parallel slab of gas with thickness L at constant T.



a. Show that looking at the slab from above, you see blackbody radiation if $\tau_{\lambda,0} \gg 1$. State any assumptions that you make.

b. If $\tau_{\lambda,0} \ll 1$, show that you see absorption lines superimposed on the spectrum of the incident radiation if $I_{\lambda,0} > S_\lambda$ and emission lines of $I_{\lambda,0} < S_\lambda$. State any assumptions that you make.

8. The disk of a galaxy can be modeled as a uniform slab of material of mass density, ρ, that is of a full thickness $2H$ in the $\hat{z}$ direction, and is effectively infinite in the $\hat{x}$ and $\hat{y}$ directions. Assume that the mass density for $z > H$ is zero.

a. Compute the effective gravity, $\vec{g}$, as a function of the distance z above the midplane both inside and outside the disk. *Hint: make use of Gauss' law for gravity* $\oint_S \vec{g} \cdot d\vec{A} = -4\pi GM$.

b. Find the speed, v_z, that a star must have starting at the middle of the disk, to get above height H (i.e., just outside the mass distribution). Express your answer in terms of ρ, G and H.

9. The equation of state for a white dwarf star is:

$$P = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

a. This equation contains no temperature term. Is this correct? Please explain.

b. The central pressure can be estimated as $P_c \approx \frac{2}{3}\pi G\rho^2 R_{WD}^2$. Assuming a constant density, $\rho = M_{WD}/\frac{4}{3}\pi R_{WD}^3$, estimate the radius of the white dwarf.

c. Sketch the relation between the radius and mass for white dwarf stars. Explain why the relationship has this form. Are white dwarf stars in hydrostatic equilibrium? Explain your answer.

10. The Rayleigh-Jean's approximation (when $\lambda \gg \frac{hc}{kT}$) is used to simplify the Planck function at some wavelength ranges. Confirm/show whether radio astronomers are justified using this approximation at radio wavelengths.