Preliminary Examination: Astronomy

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Instructions:

- Answer 8 of the 10 questions (10 points each). You must clearly identify the 8 questions you wish to be graded by circling the question number on this exam and in the answer books.
- Total time for the test is three hours
- Partial credit will be given if merited, so show your work towards completion of the problems
- Calculators (provided) are allowed. No notes are allowed – constants and equations are listed.
- *Caveat*: In equations "nu" and "vee" look similar: $\lambda\nu = c$, $E_k = \frac{1}{2}mv^2$

Speed of light	$c = 3 \times 10^8$ m s^{-1}
Planck's constant	$h = 6.626 \times 10^{-34}$ J s = 4.136×10^{-15} eV s
Planck's const. x speed of light	$hc \approx 1240$ eV nm
Gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Stefan-Boltzmann's constant	$\sigma = 5.67 \times 10^{-8}$W m^{-2} K^{-4}
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J K^{-1} = 8.62×10^{-5} eV K^{-1}
Mass of the Sun	$1\ M_\odot = 1.99 \times 10^{30}$ kg
Solar Effective Temperature	$T_\odot = 5777$ K
Luminosity of the Sun	$1\ L_\odot = 3.9 \times 10^{26}$ W
Radius of the Sun	$1\ R_\odot = 6.95 \times 10^8$ m
Radius of the Earth	$1\ R_E = 6.37 \times 10^6$ m
Mass of a hydrogen atom	$1\ M_H = 1.67 \times 10^{-27}$ kg
Bohr radius	$a_B = 0.529$ Å = 0.0529 nm
Ground state energy of H atom	$E_1 = -13.6$ eV
Astronomical unit	1 AU = 1.496×10^{11} m
Parsec	1 pc = 3.26 ly = 3.086×10^{16}m = 206,265 AU
1 eV	1.6×10^{-19} J

Useful equations:

$\lambda_{max} = 0.29/T$ cmK	Wien's Law
$F = \sigma T^4$	Stefan - Boltzmann Law
$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$	Planck Function
$P = nkT$	Ideal gas law, where $n = \rho/\mu m_H$

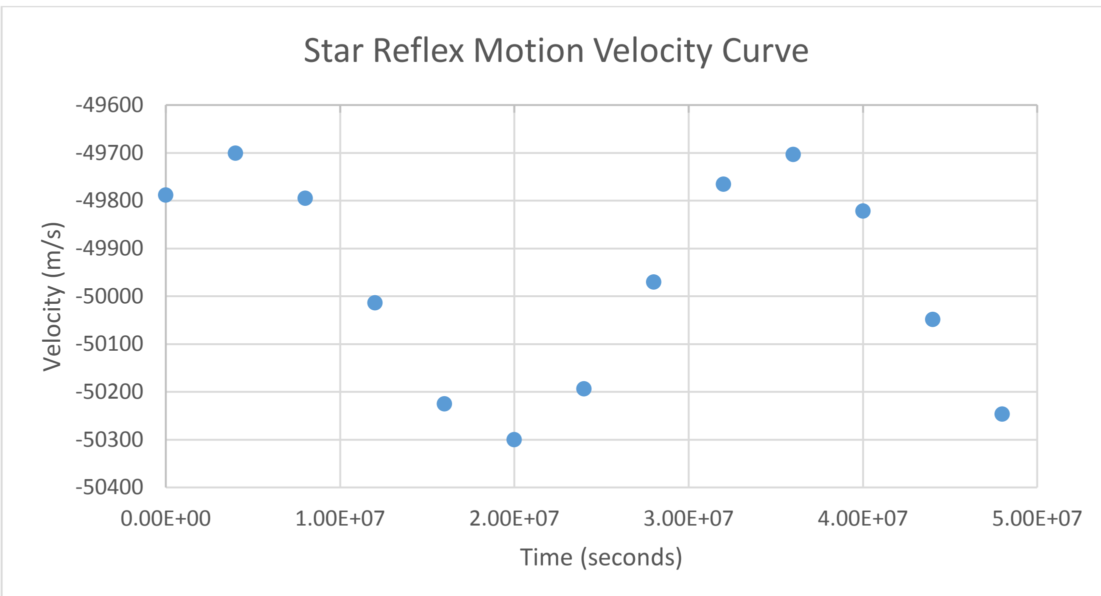
K.E. = $\frac{3}{2}kT$ (per particle) and $V = (3kT/m)^{1/2}$ (average velocity of particle)

$P_{rad} = aT^4/3$	Radiation pressure
$P_G = GM^2/4\pi R^4$	Gravitational pressure
$V = (\lambda_{obs}-\lambda_0/\lambda_0)\ c$	Doppler velocity
$S = W/(4\pi d^2 \Delta\nu)$	Flux and power
$P^2 = \frac{4\pi^2 a^3}{G(M_* + m_p)}$	Kepler III
$\nu_\infty/\nu_0 = (1 - 2GM/r_0c^2)^{1/2}$	Gravitational redshift
$R_s = 2GM/c^2$	Schwarzschild radius

1. Planets are now known to orbit many stars, and an early technique for discovering and measuring the mass of planets is to measure the orbital reflex motion of the star.

   This velocity curve is interpreted as a single planet in a circular, equatorial orbit (i = 90°) about a G2 V star very similar to our sun. (G2 V stars were the earliest targets for planetary discovery, anticipating Earth-like planets.)
   a. What is the recession velocity of the star/planet system?
   b. What is the reflex velocity amplitude of the star?
   c. Calculate the mass of the planet orbiting this star.
   d. Is this a terrestrial world?

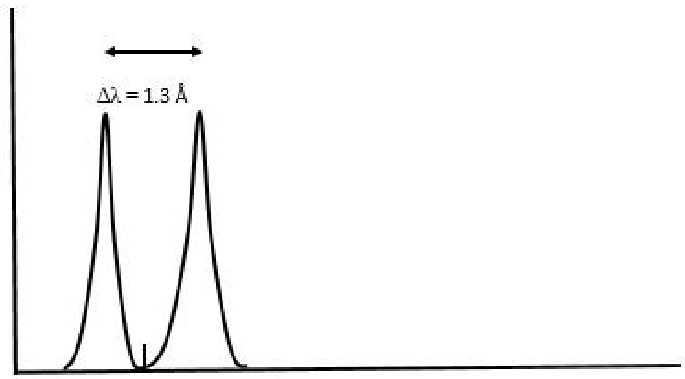
   (Caveat: Planets are a small fraction of the mass of the host star, thus simply calculating (M_{star} + m_{planet}) does not yield an accurate result.)



2. An early confirmation of special relativity derived from an experiment to measure the lifetime of muons created by cosmic rays colliding with the atoms in Earth's upper atmosphere. Muons have an average rest-frame lifetime τ = 2.2 μs. The number of muons in a sample decreases exponentially over time, i.e. $N(t) \propto e^{-t/\tau}$. The detector at the top of Mt. Washington in New Hampshire counted 563 muons per hour. These muons propagate downward with a speed u = 0.9952 c. What is the muon flux counted by the detector at Harvard, 1907 m below the one on the mountain top?

3. Quasar 3C 446 is violently variable; its luminosity at optical wavelengths has been observed to change by a factor of 40 in as little as 10 days. Using the redshift parameter z = 1.404 measured for 3C 446, determine the time for the luminosity variation as measured in the quasar's rest frame.

4. A Planetary Nebula with the form of a spherical shell, imaged with the Hubble Space Telescope over many years, has an angular radius observed to be expanding at a rate of 4.2 milli-arcsecs per year. A spectrum of the Hα line through the center of the nebula shows line splitting, as shown below, indicating an expansion velocity of 30 km/s. Assume the expansion is spherically symmetric. From this information, calculate the distance from the Sun to the Planetary Nebula in parsecs.



5. Hydrogen is the simplest and most ubiquitous atom in the universe, thus its quantum structure: size, momentum, energy levels, level transitions, etc. are broadly important and can be calculated using quasi-classical methods. The first excited state of the hydrogen atom (n = 2) has a lifetime $\tau = 10^{-8}$ s.
   a. Estimate the speed of the electron which is confined to a box of the size of the hydrogen atom.
   b. Find the wavelength λ of the photon which is emitted when the hydrogen atom transitions from the first excited state to the ground state.
   c. Find the uncertainty Δλ in the wavelength of the photon in (b), above.
   d. Find the ratio of the probabilities for the hydrogen atom to be in the first excited and ground states at the solar effective temperature $T_{\odot}$.

6. The virial theorem, $-2\langle K\rangle = \langle U\rangle$, where K is the time-averaged kinetic energy and U the potential energy of the system, is invoked to measure the mass of clusters of galaxies given the velocity dispersion of the galaxies in the cluster and the metric size of the cluster. An early indication of the concept of Dark Matter arose from Fritz Zwicky's analysis of the relatively nearby Coma cluster. Its velocity dispersion is measured to be σ = 977 km s^{-1} (modern value) and the radius of the cluster is about 3 Mpc.
   a. What is the virial mass of the cluster in units of $M_{\odot}$?
   b. Photometric measurements place the luminosity of the cluster at ~ 5 x 10^{12} $L_{\odot}$.

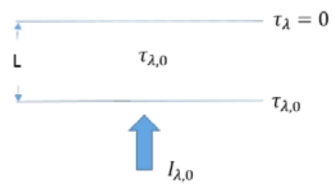
What is the mass-to-light (M/L) ratio for Coma? Make a comment about your derived value.

7. The radiative transfer equation describes the spectral energy distribution of radiation passing through matter, with optical depth τ_λ and source function S_λ

$$I_\lambda(0) = I_{\lambda,0}e^{-\tau_{\lambda,0}} - \int_{\tau_{\lambda,0}}^{0} S_\lambda\, e^{-\tau_\lambda} d\tau_\lambda$$

where $I_\lambda(0)$ is the emergent flux.

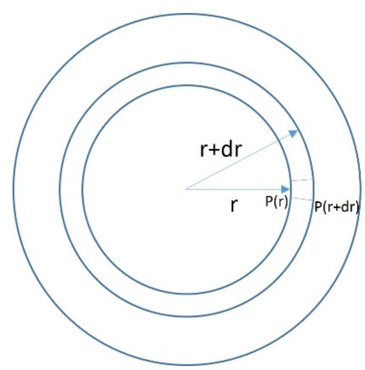
Consider a horizontal plane-parallel slab of gas with thickness L at constant T.



a. Show that looking at the slab from above one observes blackbody radiation if $\tau_{\lambda,0} \gg 1$. State any assumptions you make.
b. If $\tau_{\lambda,0} \ll 1$ show that one observes absorption lines superimposed on the spectrum of incident radiation if $I_{\lambda,0} > S_\lambda$ and emission lines if $I_{\lambda,0} < S_\lambda$.

State any assumptions you invoke.

8. The vast majority of stars are observed to be nearly perfect spheres because they are in hydrostatic equilibrium. Using the geometry of the drawing, describe the condition of hydrostatic equilibrium in a stellar interior.



Consider a spherical mass shell at radius r and thickness dr.

a. What forces cause the spherical star to be stable, and how does one describe this equilibrium state in terms of those forces? It might also be useful to consider forces on a differential mass element of unit surface area embedded in the shell.
b. Derive an equation for the mass interior to r in terms of the density at r.
c. For $\rho(r) = 1.5 \times 10^5 e^{-0.6r/R}$, where R is the stellar radius, estimate the mass of the star.

9. Comparing the distance to galaxies derived from an intrinsic physical property and comparing that to the redshift is a primary technique for calibrating the distance scale. Relatively nearby spiral galaxies that are more luminous generally have larger values of their maximum rotation velocity, typically measured as an optical or near-infrared spectral line width related to rotational dynamics in the galaxy. This is the "Tully-Fisher Relation", which is often used as a distance indicator for spiral galaxies.

   From basic physical principles, show that $L \propto V^4$ where L is a spiral's luminosity, and V is its maximum rotational velocity. Assume a constant mass to light ratio for spirals, and also constant surface brightness.

10. The WMAP mission has accurately measured the relic radiation of the creation of our universe in which we are immersed to be 2.725 K. The Sun is in motion relative to this Cosmic Microwave Background (CMB) radiation, which creates small relativistic frequency shifts of the detected CMB radiation. These can be interpreted as small shifts in the peak of the CMB blackbody energy distribution. The relativistic Doppler Shift describes these frequency shifts,

$$v_{obs} = \frac{v_{rest}\sqrt{1 - {u^2}/{c^2}}}{1 + ({u}/{c})\cos\phi}$$

   <u>where u is the velocity of the source and ϕ is the angle of motion of the source relative to the observer</u>. (E. g. if the motion is directly away from the <u>observer</u>, ϕ = 0°, and $V_r = u$ – the "usual" velocity of recession measured by a redshift, say.)

   a. Starting with this equation, show that

$$T_{moving} = \frac{T_{rest}\sqrt{1 - {v^2}/{c^2}}}{1 - {v}/{c}\cos\theta}$$

   where θ is the angle between the direction of observation and the direction of the Sun's motion.

   b. Show that for $v \ll c$,

$$T_{moving} \cong T_{rest}\left(1 + \frac{v}{c}\cos\theta\right)$$

   This equation, dependent upon the cosine of the observer's motion through the CMB, describes the lowest order term in the observed perturbations.

   c. What motions create this dipole distribution of observed temperature?