Preliminary Examination: Astronomy

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Instructions:

- Answer 8 of the 10 questions (10 points each)
- Total time for the test is three hours
- Partial credit will be given if merited, so show your work towards completion of the problems
- No notes allowed - potentially useful constants and equations are provided below
- Calculators (provided) allowed

Speed of light	$c = 3 \times 10^8\ \mathrm{m\,s^{-1}}$
Planck's constant	$h = 6.626 \times 10^{-34}\ \mathrm{J\,s} = 4.136 \times 10^{-15}\ \mathrm{eV\,s}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Stefan-Boltzmann's constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Boltzmann's constant	$k = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}} = 8.62 \times 10^{-5}\ \mathrm{eV\,K^{-1}}$
Mass of the Sun	$1\ \mathrm{M_\odot} = 1.99 \times 10^{30}$ kg
Luminosity of the Sun	$1\ \mathrm{L_\odot} = 3.8 \times 10^{26}$ W
Radius of the Sun	$1\ \mathrm{R_\odot} = 6.96 \times 10^{8}$ m
Mass of the Earth	$1\ \mathrm{M}_E = 5.97 \times 10^{24}$ kg
Radius of the Earth	$1\ \mathrm{R}_E = 6.37 \times 10^{6}$ m
Mass of a hydrogen atom	$1\ \mathrm{m}_H = 1.67 \times 10^{-27}$ kg
Astronomical unit	$1\ \mathrm{AU} = 1.496 \times 10^{11}$ m
Parsec	$1\ \mathrm{pc} = 3.26\ \mathrm{ly} = 3.086 \times 10^{16}\ \mathrm{m} = 206{,}265\ \mathrm{AU}$
1 eV	1.6×10^{-19} J
1 Jy	$10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$
Mass of a proton	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Thomson scattering cross section	$\sigma_T = 6.65 \times 10^{-29}\,\mathrm{m^{-2}}$

$I = I_0 e^{-\tau}$	Radiative transfer (absorption only, $\tau = n\sigma s$)
$\lambda_{max} = 0.0029/T\ \mathrm{m\,K}$	Wien's Law
$F = \sigma T^4$	Stefan-Boltzmann Law
$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$	Planck Function
P=nkT	Ideal gas law, where $n = \rho/\mu m_H$
KE $= \frac{3}{2}kT$ (per particle)	with the average velocity per particle $v = (3kT/m)^{1/2}$
$P_{rad} = aT^4/3$	Radiation pressure
$v = c(\lambda_{obs} - \lambda_0)/\lambda_0$	Doppler velocity
$\nu_\infty/\nu_0 = (1 - 2GM/r_0c^2)^{1/2}$	Gravitational redshift
$R_{sch} = 2GM/c^2$	Schwarzschild radius
$R_S = \left(\frac{3N}{4\pi\alpha}\right)^{1/3} n_e^{-2/3}$ cm	Strömgren radius
$R(t) = (1+z)^{-1}$	Scale factor
$T_0 = R(t)T(t)$	Scale factor effect on temperature

1. Observed spectral lines are not delta functions, and we can measure the with of a line by using either the equivalent width (W) or the full-width half maximum ($FWHM$). Describe the three main processes by which spectral lines can broaden.

2. Suppose that a hot star is losing mass in an expanding shell. As the shell moves away from the star at some velocity V, it expands and cools and can absorb radiation from the star. The star emits a Balmer series emission line at a rest wavelength of 6563 Ångströms (the H-alpha line) and a shell-absorption feature is seen 32.81 Ångströms blueward of the rest wavelength.

   a) What is the expansion velocity of the shell?

   b) The star is displaying a so called P-Cygni profile, which is a spectral line profile usually used to indicate outflows. Sketch such a profile using intensity versus velocity on a diagram, and then explain how the profile is interpreted as an outflow (use a sketch if you find it helpful).

3. Cepheid variables are important distance indicators in the extragalactic distance ladder. δ Cephei, at a distance of 300 pc from Earth, is the prototypical Cepheid variable.

   A Cepheid was detected in NGC 3351 from its variability (which called attention to the star) and its light curve. Multiple HST observations were used to measure the period, which is virtually identical to that of δ Cephei. The NGC 3351 Cepheid is measured to be 9×10^{-10} times fainter than δ Cephei, however. What is the distance to NGC 3351?

4. a. Describe what is meant by a system being in *virial equilibrium.*

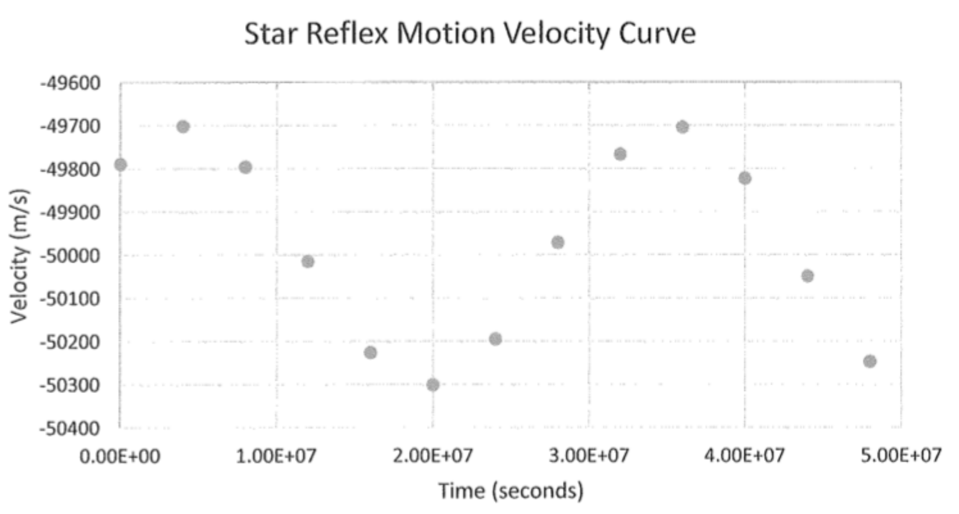
   b. NGC1316 is a giant elliptical galaxy with an approximate mass of 2×10^{12} $M_\odot$ within a radius of 400 kpc. How long would it take for a star at the edge to orbit the center once?

   c. Assume that NGC1316 has been capturing smaller satellite galaxies up til present time, and given your answer in b), would you expect the outer parts of NGC1316 to be in virial equilibrium? Why or why not?

5. Planets are now known to orbit many stars, and an early technique for discovering and measuring the mass of planets is to measure the orbital reflex motion of the star.

   The velocity curve below is interpreted as a single planet in a circular, equatorial orbit ($i = 90°$) about a G2V star very similar to our Sun.

   a) What is the recession velocity of the star/planet system?
   b) What is the reflex velocity amplitude of the star?
   c) What is the mass of the planet orbiting the star? *: Caveat: Planets are a small fraction of the mass of the host star, thus simply calculating ($M_{star} + m_{planet}$) will not give an accurate result.*
   d) Is this a terrestrial world?



6. The Big Bang theory is the theory that the Universe started off in an extremely hot, dense state, which then rapidly expanded, cooled, and became more tenuous over time. This theory requires that at some point in the past the Universe was born, that the Universe was extremely hot, and that objects were much closer together.

   State and explain the three key pieces of evidence for the Big Bang theory.

7. When a giant molecular cloud collapses to form stars, many more low-mass stars will be formed than high-mass stars. The number of stars of mass M that are formed from the collapse of a molecular cloud is denoted as $N(M)$ and is related to the stellar mass M by the Initial Mass Function (IMF). Assume that $N(M) \propto M^{-2.35}$.

   If a molecular cloud collapses and forms a globular cluster containing 10,000 stars with $M \approx 1\mathrm{M}_\odot$, how many O-type stars with $M \approx 40\mathrm{M}_\odot$ would be expected in the cluster?

8. Dust grains are ubiquitous in the interstellar medium, and affect chemistry as well as the observed properties of light. Calculate the temperature of a dust grain at 100 AU distance from a newly formed F0 main-sequence star of radius $R_* = 9.74 \times 10^8$m and effective temperature $T_{eff} = 7,300$K. Assume the grain is spherically symmetric and emits and absorbs radiation as a perfect blackbody. *Hint: Assume the grain is in thermal equilibrium, such that during a given interval of time the amount of energy absorbed by the grain equals the amount of energy radiated away.*

9. The Eddington luminosity, L_{Edd}, is defined as the maximum luminosity for which the gravitational pressure can keep the star bound ($L_{Edd} = \frac{4\pi G m_p c}{\sigma_T} M$, where σ_T is the Thomson scattering cross section and m_p is the mass of a proton). The luminosity of an accreting object comes, with some efficiency η, from the conversion of the gravitational binding energy ($L = \eta \frac{dE}{dt}$).

   a) Use this information to form a simple differential equation for how the mass of a black hole grows with time (as a function of mass of the black hole), while it is accreting at the Eddington luminosity.

   b) Assuming a radiative efficiency of $n = 0.1$, how long would it take for a black hole of $M = 7\mathrm{M}_\odot$ to reach a luminosity of 10^{40} J/s?

10. The luminosities of galaxies are difficult to measure as they are extended object with no well-defined edges. We use the *surface brightness* to compare the energy production between galaxies, defined as the amount of radiation per square arcsecond on the sky.

    Neglecting any effects of extinction and K-correction, show that the surface brightness of a galaxy is independent of the distance to the observer.