Rolling v/s Sliding

<u>Solution:</u>

The correct answer is c.)

For the weight that rolls, energy conservation yields:

$$mgh = \frac{1}{2}\left(mv_{rolling}^2 + I\omega^2\right) \Rightarrow mv_{rolling}^2 = 2mgh - I\omega^2$$

$$\Rightarrow v_{rolling}^2 = 2gh - \left(\frac{I}{m}\right)\left(\frac{v_{rolling}}{r}\right)^2 \Rightarrow v_{rolling}^2\left[1 + \left(\frac{I}{mr^2}\right)\right] = 2gh$$

$$\therefore, v_{rolling} = \sqrt{\frac{2gh}{1 + \left(I/mr^2\right)}}$$

Rolling v/s Sliding

In the case of the weight that slides, *all* of its initial (stored) potential energy is converted to translational kinetic energy:

$$\cancel{m}gh = \left(\frac{1}{2}\right)\cancel{m}v^2_{sliding} \Rightarrow v_{sliding} = \sqrt{2gh}$$

Clearly, the speed of the weight that slides is greater, since:

$$\left(\frac{I}{mr^2}\right) > 0 \Rightarrow v_{rolling} < \sqrt{2gh}$$

Note that we do not require the exact expression for the moment of inertia (I) of the slotted weights, to make this prediction!