Effect of Friction

Solution:

The correct answer is a.)

Energy Conservation with friction (which causes part of the potential energy to be converted to rotational K.E.) taken into account yields:

$$mgh = \left(\frac{1}{2}\right)mv_b^2 + \left(\frac{1}{2}\right)I\omega_b^2 \qquad \dots (1)$$

For a solid sphere, moment of inertia about an axis passing through its center:

$$I = \left(\frac{2}{5}\right) mr^2 \qquad \dots (2)$$

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Also, friction causes the linear and angular speeds to be related linearly:

$$\omega = \left(\frac{v}{r}\right) \qquad \dots (3)$$

Substituting (2) and (3) into (1), we get:

$$mgh = \left(\frac{1}{2}\right) \left[mv_b^2 + \left(\frac{2}{5}mr^2\right)\left(\frac{v_b}{r}\right)^2\right]$$

$$\Rightarrow mgh = \left(\frac{1}{2}\right)mv_b^2 \left[1 + \left(\frac{2}{5}\right)r^2\left(\frac{1}{r^2}\right)\right] \Rightarrow \left(\frac{7}{10}\right)v_b^2 = gh$$

$$\dots (4)$$