Horizontal Range

Solution:

The correct answer is d.)

We know from Question 2, that the total time of flight of a projectile with initial speed v_0 launched at an angle θ to the horizontal is

$$T = \left(\frac{2v_{0y}}{g}\right) = \left(\frac{2v_0 \sin\theta}{g}\right)$$

Horizontal Range

Since horizontal range refers to the horizontal displacement of the projectile, we simply multiply the horizontal component of speed by the total time of flight to get the horizontal distance covered in that time:

$$\Rightarrow R = v_{0x}T = (v_0 \cos\theta)\left(\frac{2v_0 \sin\theta}{g}\right)$$

$$\Rightarrow R = \left(\frac{v_0^2}{g}\right)(2\sin\theta\cos\theta) = \left(\frac{v_0^2 \sin(2\theta)}{g}\right)$$

Horizontal Range

Note that this expression for R could have also been obtained by eliminating t between the kinematic equations in the horizontal and vertical directions:

$$R = \left(v_{0x}\right)t + \left(\frac{1}{2}\right)at^2 = \left(v_0 \cos\theta\right)t$$

and,

$$S_y = \left(v_0 \sin\theta\right)t + \left(\frac{1}{2}\right)\left(-g\right)t^2 \Rightarrow 0 = \left(v_0 \sin\theta\right)t - \left(\frac{1}{2}\right)gt^2$$

However, this expression for R is correct, only when the projectile's range is measured at the same height as its initial (launch) height.

Horizontal Range

If this were not the case, so that the ball had been shot from an initial height h above the floor, and landed on a target placed at a height h_0 above the floor ($h_0 < h$), the time of flight would had to have been found by solving the quadratic:

$$\left(h - h_0\right) = v_{0y}t + \left(\frac{1}{2}\right)\left(-g\right)t^2 = \left(v_0 \sin\theta\right)t - \left(\frac{1}{2}\right)gt^2$$

for t. The horizontal range would then be found as:

$$R = \left(v_{0x}\right)t = \left(v_0 \cos\theta\right)t$$