The time lens

The time equivalent of propagation distance is diffraction. To a propagation distance $L$, we substitute the quantity $L \frac{d^2 k}{d\Omega^2}$ (1)

where $k(\Omega)$ is the wave vector of the light in the medium where the light propagates. The dispersion of the medium $\frac{d^2 k}{d\Omega^2}$ is generally a small quantity. There are optical arrangements (pairs of gratings or pairs of prisms) for which Eq. (1) still holds, but the quantity $\frac{d^2 k}{d\Omega^2}$ is much larger than for an homogeneous medium [3]. The property of a lens is to create a parabolic wavefront in space. The time equivalent is a device that creates a parabolic phase modulation in time. The temporal focal distance is:

$$f_T = \left( \frac{\partial^2 \varphi}{\partial t^2} \right)^{-1}$$ (2)

where $\varphi$ is the phase factor of the complex electric field of the light. The classical lens formula, which in space is:

$$\frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{f}$$ (3)

becomes in time:

$$\left( L_1 \frac{d^2 k_1}{d\Omega^2} \right)^{-1} + \left( L_2 \frac{d^2 k_2}{d\Omega^2} \right)^{-1} = (f_T)^{-1}$$ (4)

where $L_{1,2}(\frac{d^2 k_{1,2}}{d\omega^2})$ are the dispersion characteristics of the object and image side, respectively.

As in optical imaging, to achieve large magnification with practical devices, short focal lengths are desired. For time imaging this translates into a short focal time $f_T$ which in turn requires a suitably large phase modulation. One possible approach to create a large phase modulation is cross-phase modulation, in which a properly shaped powerful “pump” pulse creates a large index sweep (quadratic with time) in the material of the “time lens”. Another approach is to use sum or difference frequency generation to impart the linear chirp of one pulse into the pulse to be “imaged”. The linear chirp can be obtained by propagating of a strong pulse through a fiber. A detailed
Figure 1: Space–time analogy. (a) Spatial imaging configuration. The “object” is a graphic representation of a succession of a three pulse sequence. The “real image” show a magnified, inverted picture. (b) the temporal imaging configuration. A pair of gratings provides the dispersion analogue to the distances object lens $d_1$ and lens-image $d_2$. The object is a three pulse sequence. The “image” is a reversed, expanded three pulse sequence. A time lens that can provide the temporal quadratic phase modulation is sketched in (c). The sum or difference frequency of the dispersed signal with a linearly chirped pulse (having preferably a square envelope) is made in a nonlinear crystal. The output envelope is proportional to the product of the two signals being mixed, and has thus a linear chirp imparted on the signal.
review of this “parametric temporal imaging” can be found in refs. [4, 5]. A sketch of the technique, with its spatial analogy, is shown in Fig. 1.

The elements of analogy between classical optics and short pulse propagation are summarized in table 1.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Focal length</th>
<th>Lens formula</th>
<th>Magnification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space</td>
<td>L</td>
<td>( \frac{1}{l_1} + \frac{1}{l_2} = \frac{1}{f} )</td>
<td>( M = \frac{l_1}{l_2} )</td>
</tr>
<tr>
<td>Time</td>
<td>( L \frac{d^2k}{dT^2} )</td>
<td>( \left(L \frac{d^2k_1}{dT^2}\right)^{-1} + \left(L \frac{d^2k_2}{dT^2}\right)^{-1} = (ft)^{-1} )</td>
<td>( \frac{L_1 \frac{d^2k_1}{dT^2}}{L_2 \frac{d^2k_2}{dT^2}} )</td>
</tr>
</tbody>
</table>

Table 1: summary table comparing space and time lenses
References


