HW 19 Solutions (Partial)

19-16. How can a 480Ω resistor be converted to 320Ω?

To lower its effective resistance, we can add a second resistor in parallel. The value of the second
resistor should be:

\[
\frac{1}{320}\Omega = \frac{1}{480\Omega} + \frac{1}{R_2} \implies \frac{1}{32} - \frac{1}{48} = \frac{1}{R_2}
\]

\[
\frac{3}{96} - \frac{2}{96} = \frac{1}{R_2} \implies \frac{1}{R_2} = \frac{1}{96}\Omega
\]

19-19.

b) What happens to each current?
Having solved part a), this is trivial - since the \(R_n\) do not change, the only way \(I_n\) can change is if \(V_I\) change (\(R = \frac{V}{I}\)).
So the change in \(V\) are simply mirrored in \(I\): \(I_1\) and \(I_2\) increase, \(I_3\) and \(I_4\) decrease.

C) Power output?
Since \(R_{\text{total}}\) decreases, \(I_{\text{total}}\) increases and \(P = VI\) also increase.

2) What happens to the voltages when \(S\) is closed?

Closing \(S\) adds \(R_2\) in parallel with \(R_3 + R_4\), lowering the resistance of that part of the

circuit. If that branch has been \(R_{eq}\), then the voltage drop across \(R_1\) will have to increase
to compensate. \(V_2\) will obviously increase (from zero!) and since \(R_3\) and \(R_4\) are now
sharing current with \(R_2\), \(V_3\) and \(V_4\) will decrease (as \(I_3\) and \(I_4\) do.)
d) Let $R_1 = R_2 = R_3 = R_4 = 125 \Omega$ and $V = 22.0 \text{V}$. Find all currents before and after closing the switch $S$. 

**Before**

$$R_{\text{total}} = R_1 + \frac{1}{R_3} + \frac{1}{R_4} = 125 + \frac{125}{2} = 187.5 \Omega$$

$$I_{\text{total}} = \frac{22.0 \text{V}}{187.5 \Omega} = 0.117 \text{A}$$

$I_{\text{total}}$ is the current from the battery, so it is also the current through $R_1$, i.e.

Since $R_3 = R_4$, the current will split equally between them:

$I_2 = 0$ because $R_2$ is not connected to a voltage.

**After**

$$R_{\text{total}} = 125 + \frac{1}{\frac{1}{125} + \frac{1}{125} + \frac{1}{125}} = 166.7 \Omega$$

$$I_{\text{total}} = I_1 = \frac{22.0 \text{V}}{166.7 \Omega} = 0.132 \text{A}$$

The current splits equally among 3 resistors this time:

$I_2 = I_3 = I_4 = \frac{I_1}{3} = 0.044 \text{A}$
19-23 | Show that $\Delta V = 0$ for the circuit below.

\[
\begin{align*}
\text{\text{R}}_\text{total} &= 2.0\Omega + 12.0\Omega + 8.0\Omega \\
&= 22.0\Omega \\
\text{I}_{\text{total}} &= \frac{V}{R} = \frac{12.0V}{22.0\Omega} = 0.545\text{A} \\
\Delta V &= \varepsilon - \text{I}_1 \cdot R + \varepsilon V \\
&= -8.0\Omega \cdot (0.545\text{A}) - 12.0\Omega \cdot (0.545\text{A}) \\
&\quad -2.0\Omega \cdot (0.545\text{A}) + 12.0V \\
&= 4.34\text{V} - 6.55\text{V} - 1.09\text{V} + 12\text{V} \\
&= 12.0\text{V} + 12.0\text{V} = 0
\end{align*}
\]

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19-32 | Determine $I_1, I_2$, and $I_3$, assuming $r = 10\Omega$.

\[
\begin{align*}
\text{Junction rule: } I_{\text{in}} &= I_{\text{out}} \\
I_1 + I_2 &= I_1 \\
\text{Loop Rules:} \\
\text{Upper loop:} \\
12 - 11I_1 - 8I_1 + 12 - 2I_2 - 10I_2 - 12I_1 &= 0 \\
24 - 21I_1 - 11I_2 &= 0 \\
\text{Lower loop:} \\
12 - 1I_2 - 10I_2 + 18I_3 + I_3 - 6V + 15I_3 &= 0 \\
6 - 11I_2 + 34I_3 &= 0 \\
6 + 34I_3 &= 11I_2 \\
I_2 &= \frac{6 + 34I_3}{11} \\
\Rightarrow I_2 &= \frac{6 + \frac{34}{11}I_3}{11} \\
I_3 &= \frac{18 - 21I_1}{34} \\
\text{Multiply both sides by } 11.34 &= 37.4
\end{align*}
\]
19-32 Cont’d

\[ 264 + 60I_2 - 714I_1 + 198 - 231I_1 = 374I_1 \]

\[ 1014 = 1319I_1 \]

\[ I_1 = \frac{1014}{1319} \approx 0.767 \text{ A} \]

\[ I_3 = \frac{18}{34} - \frac{21}{34} I_1 = \frac{18}{34} - \frac{21294}{44846} \]

\[ = \frac{23742}{44846} = \frac{21294}{44846} = \frac{2448}{44846} = \frac{34.72}{341.1319} = \frac{72}{1319} \approx 0.0546 \times 10^{-2} \text{ A} \]

\[ I_2 = I_1 - I_3 = \frac{1014}{1319} - \frac{72}{1319} = \frac{942}{1319} \approx 0.714 \text{ A} \]

b) What is the terminal voltage of the 6V battery?

\[ V_{AB} = E - I_2 |R| = 6V - I_2 (15\Omega) \]

\[ = 5.95 \text{ V} \]