PHYS 262: MATTER WAVES, CHAPTER 39

IN 1924, LOUIS deBROGLIE MADE A DARING ASSUMPTION. IF LIGHT COULD HAVE PARTICLE-LIKE BEHAVIOR (THE PHOTON), MAYBE PARTICLES (ELECTRONS, PROTONS, etc.) COULD HAVE WAVE-LIKE BEHAVIOR.

EINSTEIN HAD DERIVED AN EQUATION FOR THE PHOTON'S MOMENTUM USING HIS EQUATION $E^2 = (pc)^2 + (M_0c^2)^2$. PHOTONS HAVE NO MASS $\Rightarrow M_0 = 0$ $\Rightarrow E = pc$.

$$E = \frac{hc}{\lambda} \Rightarrow \frac{hc}{\lambda} = pc \quad \Rightarrow \lambda = \frac{h}{p}.$$

deBROGLIE MADE THE SAME ASSUMPTION FOR MASSIVE PARTICLES (WHICH HE CALLED MATTER WAVES).

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{MV}}$$

(FOR RELATIVISTIC SPEEDS, WE USE $p = \gamma MV$)

λ = deBROGLIE WAVELENGTH

EXAMPLE: WHAT IS THE deBROGLIE WAVELENGTH OF A 60Kg MAN WALKING WITH A SPEED OF 3mph = 1.34m/s.

$$\lambda = \frac{6.63\times10^{-34}\,J\cdot s}{(60Kg)(1.34m/s)} = 8.25\times10^{-36}\,m = 1.6\times10^{-25}\,a_0 \quad (a_0 = .5\text{Å})$$

$$\text{UNIT: } \frac{Js}{Kg\,m/s} = \frac{Kg\,m^2/s^2\,s}{Kg\,m/s} = m$$

FOR MACROSCOPIC OBJECTS, WAVE BEHAVIOR IS UNOTICEABLE!

deBroglie's Assumption Explains Bohr's Quantization

$$L = n\hbar$$

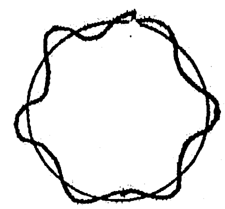
$L = Mvr = pr$. $\quad n\hbar = \frac{nh}{2\pi} \Rightarrow pr = \frac{nh}{2\pi} \Rightarrow 2\pi r = \frac{nh}{p}$

$\Rightarrow 2\pi r = n\lambda$.

ELECTRONS ORBIT THE NUCLEUS ONLY AT PLACES WHERE THE CIRCUMFERENCE IS A MULTIPLE OF THEIR deBROGLIE WAVELENGTH.

REMEMBER CONSTRUCTIVE INTERFERENCE: $Z_1 - Z_2 = m\lambda$.

FOR A WAVE GOING IN A CIRCLE $Z_1 - Z_2 = 2\pi r$



$Z_1 - Z_2$ = PATH DISTANCE.

IF ENDS DON'T MATCH UP THEN DESTRUCTIVE INTERFERENCE $\Rightarrow$ NO ELECTRON.

NOTE: THIS DRAWING IS <u>NOT</u> MEANT TO SUGGEST THAT THE ELECTRON IS MOVING IN A CIRCULAR PATTERN AROUND THE NUCLEUS.

<u>EXAMPLE</u> WHAT IS deBROGLIE WAVELENGTH FOR AN ELECTRON IN THE $n=1$ HYDROGEN ATOM.

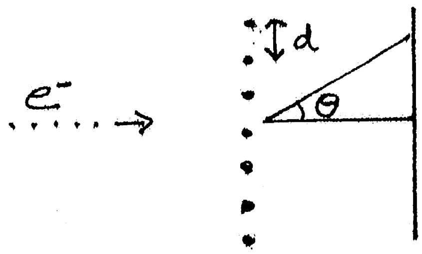
BOHR MODEL: $V = \frac{2.18 \times 10^6 \text{ m/s}}{n}$. $\lambda = \frac{h}{MV} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(2.18 \times 10^6 \text{ m/s})} = 3 \times 10^{-10} \text{ m} = 3 \text{Å}$.

REMEMBERING THAT $a_0 = .5\text{Å}$ $\Rightarrow$ WHEN λ IS COMPARABLE TO THE SCALE OF THE PROBLEM, WE WILL OBSERVE WAVE EFFECTS.

deBROGLIE'S MATTER WAVE HYPOTHESIS WAS VERIFIED IN 1927 WHEN DAVISSON AND GERMER (U.S.!) OBSERVED ELECTRON DIFFRACTION.

THEY SENT A BEAM OF ELECTRONS THROUGH A THIN SHEET OF NICKEL.

NICKEL ATOMS ARE EQUALLY SPACED.



WHEN SPACING d IS SMALL COMPARED TO $\lambda = \frac{h}{p}$,
WE GET REGIONS OF CONSTRUCTIVE AND DESTRUCTIVE
INTERFERENCE.
CONSTRUCTIVE INTERFERENCE OCCURS WHERE

$d \sin\theta = \frac{mh}{p} = m\lambda$ → EXACT SAME FORMULA AS LIGHT.

CONSEQUENCES OF MATTER WAVES

PARTICLES ARE OBJECTS WITH DEFINITE (SINGLE VALUE) POSITIONS.
WAVES ARE SPREAD OUT OVER SPACE.

PARTICLE - SINGLE
VALUE OF POSITION

MATTER WAVE - NO DEFINITE POSITION

ALL WE CAN GIVE FOR A MATTER WAVE IS THE AVERAGE VALUE OF
ITS POSITION (OR WE CAN GIVE THE MOST LIKELY POSITION).

FOR EXAMPLE, IN THE BOHR MODEL WE FOUND THAT FOR $n=1$,
$r = .5Å$. WHEN WE ADMIT THAT ELECTRONS ARE MATTER WAVES,
WE NOW HAVE TO SAY THAT $r = .5Å$ IS THE MOST LIKELY PLACE
TO FIND THE ELECTRON. THERE IS A PROBABILITY (VERY SMALL IN
PLACES) TO FIND THE ELECTRON EVERYWHERE IN THE UNIVERSE.

NOTE: STRANGE THING - WHEN MEASURING AN ELECTRON'S POSITION, WE
EITHER FIND THE WHOLE THING OR NONE OF IT. EVEN THOUGH, IT'S THIS
"FUZZY" MATTER WAVE, YOU'LL NEVER COME ACROSS PART OF AN
ELECTRON.

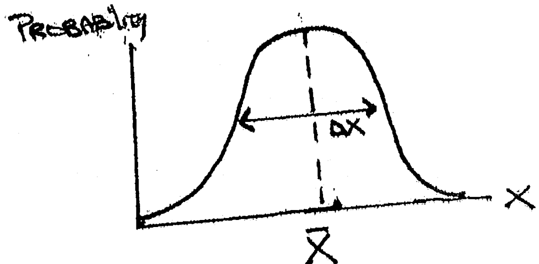
HEISENBERG UNCERTAINTY RELATION - WERNER HEISENBERG FOUND THAT IT IS IMPOSSIBLE TO PRECISELY KNOW A MATTER WAVE'S POSITION AND MOMENTUM AT THE SAME TIME.

TO MEASURE HOW WELL KNOWN A QUANTITY IS WE USE THE STANDARD DEVIATION, ΔX.

$\bar{X}$ = AVERAGE VALUE OF X.

$(\Delta X)^2 = \overline{(X-\bar{X})^2}$ → THE AVERAGE VALUE OF THE DISTANCE FROM $\bar{X}$ SQUARED.

EXAMPLE - BELL CURVE (AKA THE GAUSSIAN DISTRIBUTION).



ΔX IS A MEASURE OF HOW SPREAD OUT THE PROBABILITY IS. A LARGE $\Delta X \Rightarrow$ THE VARIABLE IS NOT VERY WELL KNOWN, IT HAS A LARGE UNCERTAINTY.

UNCERTAINTY RELATION : $\boxed{\Delta X \cdot \Delta p_x \geq \hbar}$ $\hbar = 1.05\times10^{-34}\,J\cdot s = 6.56\times10^{-16}\,eV\cdot s$

<u>EXAMPLE</u>: WHAT IS THE MINIMUM UNCERTAINTY IN MOMENTUM FOR AN ELECTRON RESTRICTED TO THE NUCLEUS.

THE NUCLEUS HAS A DIAMETER APPROXIMATELY $10^{-14}m$.

$\Rightarrow \Delta X = 10^{-14}m$.

MINIMUM UNCERTAINTY OCCURS WHEN $\Delta X \Delta p_x = \hbar \Rightarrow \Delta p_x = \frac{\hbar}{\Delta X}$

$\Rightarrow \Delta p_x = \frac{1.05\times10^{-34}Js}{10^{-14}m} = 1.05\times10^{-20}\,Kgm/s \Rightarrow p_x = 1.05\times10^{-20}\,Kgm/s$ OR BIGGER!

- WHAT ENERGY WOULD THIS ELECTRON HAVE FROM THIS MOMENTUM?

$E^2 = (pc)^2 + (M_oc^2)^2 \Rightarrow E = 3.15\times10^{-12}J$, $K = E - M_oC^2 = 3.07\times10^{-12}J = 1.9\times10^{7}eV$

THIS IS ABOUT 1.4 MILLION TIMES $E = 13.6eV$ → ELECTRONS CANNOT BE CONFINED TO THE NUCLEUS (FOR LONG).

HEISENBERG ALSO FOUND THAT THE SAME RELATIONSHIP EXISTS BETWEEN UNCERTAINTY IN ENERGY AND TIME:

$$\boxed{\Delta E \cdot \Delta t \geq \hbar}$$

EXAMPLE FOR HOW LONG COULD AN ELECTRON BE CONFINED to the NUCLEUS?

FOR THE ELECTRON TO BE IN THE NUCLEUS, ITS ENERGY MUST BE AT LEAST $3.15\times10^{-12}J \Rightarrow \Delta E = 3.15\times10^{-12}J$. SMALLEST TIME, $\Delta E \cdot \Delta t = \hbar$

$$\Rightarrow \Delta t = \frac{1.05\times10^{-34}J s}{3.15\times10^{-12}J} = 3.33\times10^{-23}s$$

$\rightarrow$ IF TIME WAS LESS THAN THIS, WE WOULD BE UNABLE TO DETERMINE IT HAD BEEN THERE.

EXAMPLE VIRTUAL PARTICLES ARE THOSE CREATED WHEN UNCERTAINTIES IN ENERGY (CALLED VACUUM FLUCTUATIONS) BECOME LARGE ENOUGH TO CREATE MATTER/ANTI-MATTER PAIRS. HOW LONG COULD A VIRTUAL ELECTRON/POSITRON (ITS ANTI-MATTER EQUIVALENT) EXIST?

TO CREATE THE PAIR REQUIRES ENERGY $E = 2M_0c^2$ (THIS ASSUMES THEY ARE STATIONARY)

$$\Rightarrow E = 2(9.11\times10^{-31}Kg)(3\times10^{8}m/s)^2 = 1.64\times10^{-13}J$$

$\Rightarrow$ MINIMUM UNCERTAINTY IS THE SAME $\Delta E = 1.64\times10^{-13}J$

$$\Rightarrow \Delta t = \frac{1.05\times10^{-34}J\cdot s}{1.64\times10^{-13}J} = 6.4\times10^{-22}s$$