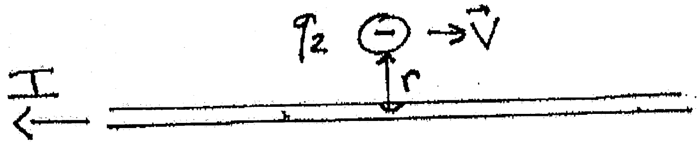
PHYS 262: RELATIVISTIC ELECTRODYNAMICS

THE STUDY OF RELATIVISTIC ELECTRIC AND MAGNETIC FIELDS IS (OF COURSE) COMPLICATED. WE WILL ONLY BE CONCERNED WITH SHOWING HOW RELATIVITY ALLOWS ELECTRIC FIELDS TO TRANSFORM INTO MAGNETIC FIELDS.

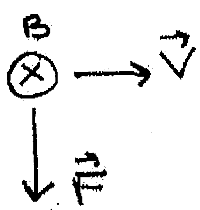
THE SIMPLEST WAY TO DO THIS IS TO FIND THE FORCE ON A MOVING CHARGED PARTICLE NEAR A WIRE.



$$\vec{F} = q_2 \vec{V} \times \vec{B}$$

FOR A WIRE $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi(1\times10^{-7}) I}{2\pi r} = \frac{(2\times10^{-7}) I}{r}$

ABOVE THE WIRE THE FIELD POINTS INTO THE PAGE (RIGHT HAND RULE)

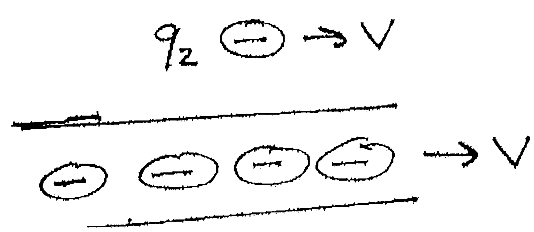


$$\Rightarrow F = |q_2| V B \sin 90° = |q_2| V B = |q_2| V \frac{(2\times10^{-7}) I}{r}$$

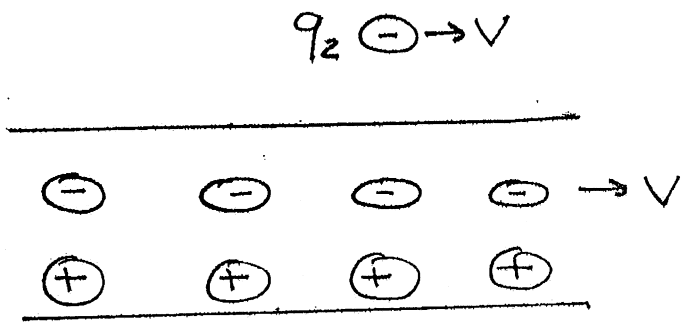
$\vec{F}$ POINTS DOWN BECAUSE $q_2 < 0$.

THE MAGNETIC FIELD IS CREATED BY THE MOVEMENT OF ELECTRONS. BECAUSE CURRENT IS DEFINED AS THE MOTION OF POSITIVE CHARGES, THE ELECTRONS ARE MOVING IN THE SAME DIRECTION AS q_2.

TO MAKE THINGS MUCH SIMPLER, LET'S ASSUME q_2 HAS THE SAME VELOCITY AS THE ELECTRONS:



FOR THE WIRE TO CREATE ONLY A MAGNETIC FIELD (AND NOT AN ELECTRIC FIELD), THE NET CHARGE ON THE WIRE MUST BE ZERO. SO EVERY NEGATIVE CHARGE MUST BE BALANCED BY A POSITIVE CHARGE. THE POSITIVE CHARGES AREN'T MOVING (OTHERWISE NO NET CURRENT), BUT BY HAVING THE SAME SPACING BETWEEN THE POSITIVE AS BETWEEN THE NEGATIVE, THE NET CHARGE WILL BE ZERO.



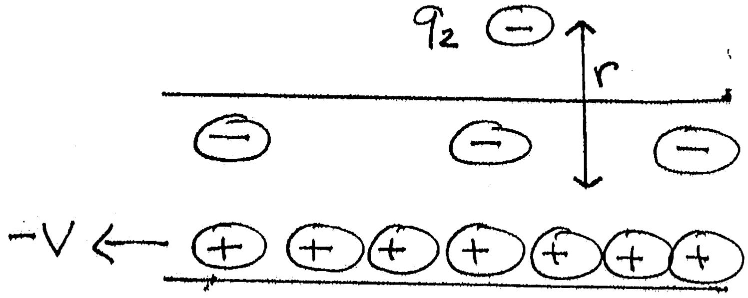
FRAME 1

$E = 0, \quad F = \dfrac{|q_2| V \mu_0 I}{2\pi r}$

LET'S LOOK AT ANOTHER INERTIAL FRAME, FRAME 2. THIS WILL BE THE FRAME WHERE THE NEGATIVE CHARGES AND q_2 ARE AT REST. THIS MEANS THAT THE POSITIVE CHARGES WILL BE MOVING IN THE OPPOSITE DIRECTION (LEFT) WITH A VELOCITY $-V$.

IN FRAME 1, WE MEASURED THE PROPER LENGTH FOR THE DISTANCE BETWEEN THE POSITIVE CHARGES. HOWEVER, WE MEASURED THE CONTRACTED LENGTH FOR THE DISTANCE BETWEEN THE NEGATIVE CHARGES.

$\Rightarrow$ IN FRAME 2, THE NEGATIVE CHARGES ARE FARTHER APART WHILE THE POSITIVE CHARGES ARE CLOSER TOGETHER.



FRAME 2

$E \neq 0$

THE ELECTRIC FIELD OF A WIRE:

$$E = \frac{2K\lambda}{r} \quad . \quad K = \frac{1}{4\pi\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$\lambda = \frac{dq}{d\ell} \rightarrow$ CHARGE PER LENGTH, i.e., CHARGE DENSITY

LET λ_0 BE THE CHARGE DENSITY FOR THE POSITIVE CHARGES IN FRAME 1.

THE NET CHARGE DENSITY IN FRAME 1 IS $\lambda_1 = \lambda_0 + \lambda_{1,-}$.

$\lambda_{1,-}$ = NEGATIVE CHARGE DENSITY. EQUAL SPACING $\Rightarrow \lambda_{1,-} = -\lambda_0$

$\Rightarrow \lambda_1 = 0 \Rightarrow E_1 = 0$ (AS REQUIRED).

IN FRAME 2: THE NEGATIVE CHARGES ARE FARTHER APART BY A FACTOR OF $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ * $\Rightarrow \lambda_{2,-} = \frac{-\lambda_0}{\gamma}$ (* $L_0 = L\gamma$)

THE POSITIVE CHARGES ARE CLOSER BY A FACTOR OF γ

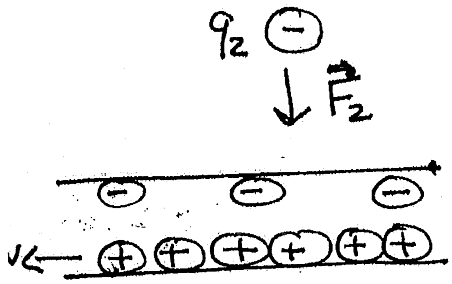
$\Rightarrow \lambda_{2,+} = \lambda_0 \gamma$

$\Rightarrow \lambda_2 = \lambda_{2,+} + \lambda_{2,-} = \lambda_0 \left(\gamma - \frac{1}{\gamma}\right)$

$$E_2 = \frac{\lambda_0}{2\pi\epsilon_0 r}\left(\gamma - \frac{1}{\gamma}\right)$$

NOTICE THAT r IS NOT CHANGED (LENGTH CONTRACTED) BECAUSE IT IS PERPENDICULAR TO $\vec{V}$.

THE FORCE ON q_2 IN FRAME 2 IS GIVEN BY THE ELECTRIC FORCE, $F_2 = |q_2| E_2$. BECAUSE THE WIRE IS OVERALL POSITIVELY CHARGED AND q_2 IS NEGATIVE, THE ELECTRIC FORCE IS ATTRACTIVE.



CALL THE DIRECTION OF THE FORCE, y.

$$F_2 = |q_2| E_2 = |q_2| \frac{\lambda_0}{2\pi\epsilon_0 r}\left(\gamma - \frac{1}{\gamma}\right)$$

NOTE: THERE IS A MAGNETIC FIELD CREATED BY THE MOVING POSITIVE CHARGES; HOWEVER, q_2 DOES NOT FEEL A FORCE FROM IT BECAUSE IT IS STATIONARY IN FRAME 2.

WE NEED F_1 = FORCE ON q_2 IN FRAME 1.

SKIPPING SOME CONFUSING DETAILS ABOUT RELATIVISTIC FORCES

IF $F_1 = M\frac{d^2y_1}{dt_1^2}$ THEN $F_2 = M\frac{d^2y_2}{dt_2^2}$

USE THE LORENTZ TRANSFORMATION WITH FRAME 1 BEING S AND FRAME 2 BEING S'.

$$\begin{aligned} y &= y' \\ t &= \gamma\left(t' + \frac{x'u}{c^2}\right) \end{aligned} \quad \Rightarrow \quad \begin{aligned} y_1 &= y_2 \\ t_1 &= \gamma\left(t_2 + \frac{x_2 V}{c^2}\right) \end{aligned}$$

IN FRAME 2, q_2 WILL ACCELERATE STRAIGHT DOWN $\Rightarrow$ NO CHANGE IN X_2 POSITION $\Rightarrow$

$$dt_1 = \gamma dt_2 \Rightarrow d^2t_1 = \gamma d^2t_2 .$$

$$d^2y_1 = d^2y_2$$

$$\Rightarrow F_1 = M\frac{d^2y_1}{d^2t_1} = M\frac{d^2y_2}{\gamma d^2t_2} = \frac{1}{\gamma}F_2$$

$$\Rightarrow F_1 = \frac{1}{\gamma}\frac{|q_2|\lambda_0}{2\pi\epsilon_0 r}\left(\gamma - \frac{1}{\gamma}\right) = \frac{|q_2|\lambda_0}{2\pi\epsilon_0 r}\left(1 - \frac{1}{\gamma^2}\right)$$

$$1 - \frac{1}{\gamma^2} = 1 - \frac{1}{\frac{1}{1 - v^2/c^2}} = 1 - (1 - v^2/c^2) = v^2/c^2$$

$$\Rightarrow F_1 = \frac{|q_2|\lambda_0}{2\pi\epsilon_0 r}\frac{v^2}{c^2} = |q_2| v \left(\frac{\lambda_0 v}{2\pi\epsilon_0 r c^2}\right) \qquad \frac{1}{c^2} = \mu_0\epsilon_0$$

$$\Rightarrow F_1 = |q_2| v\left(\frac{\mu_0 \lambda_0 v}{2\pi r}\right) \qquad \lambda_0 v = \frac{dq}{d\ell}\cdot\frac{d\ell}{dt} = \frac{dq}{dt} = I$$

$$\Rightarrow F_1 = |q_2| v \left(\frac{\mu_0 I}{2\pi r}\right) = |q_2| v B\,!$$