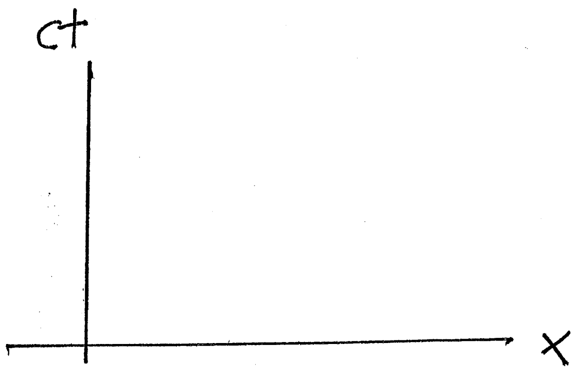
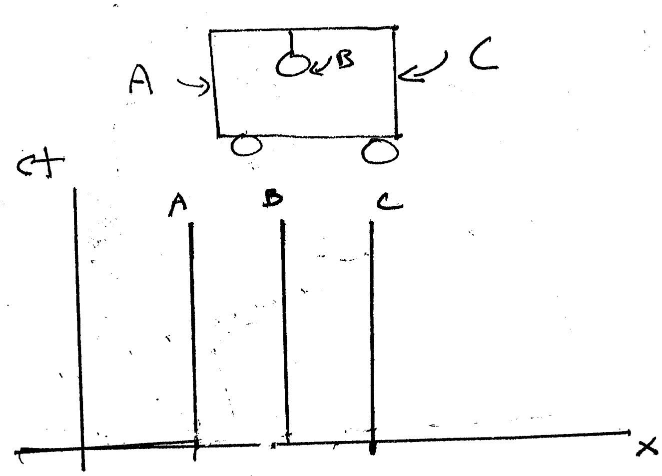
SPACE-TIME DIAGRAMS

A WAY OF "SEEING" WHAT IS GOING ON IN RELATIVITY IS BY DRAWING PICTURES OF ~~pos~~ c× TIME VS. POSITION = SPACE-TIME AKA OR MINKOWSKI DIAGRAMS.

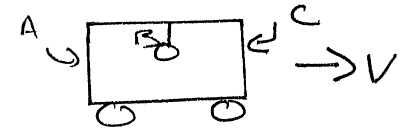


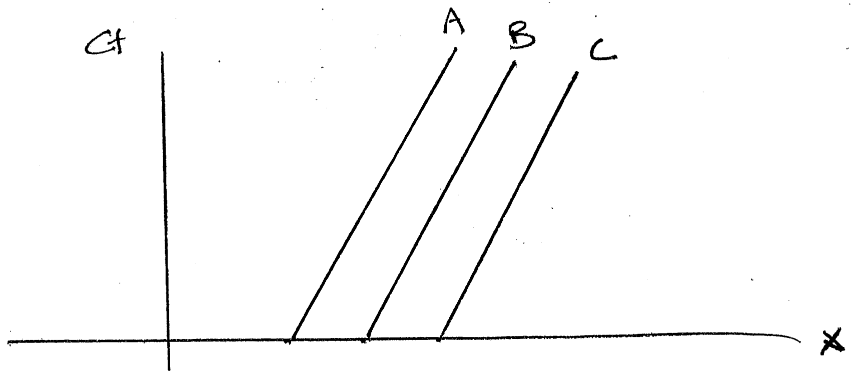
WORLD LINE = CURVE DRAWN ON A SPACE-TIME DIAGRAM

EXAMPLE → RAILROAD CAR AT REST



RAILROAD CAR MOVING WITH SPEED V

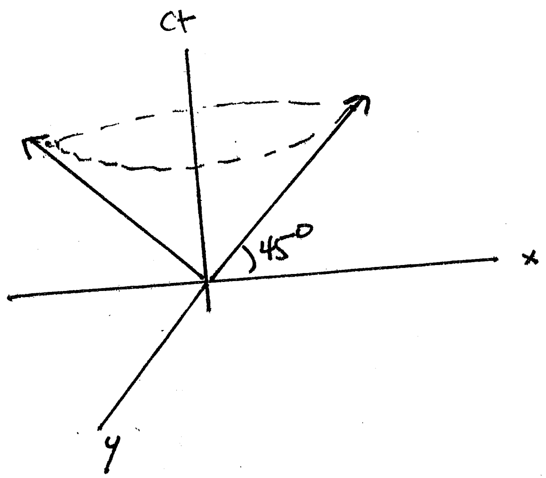




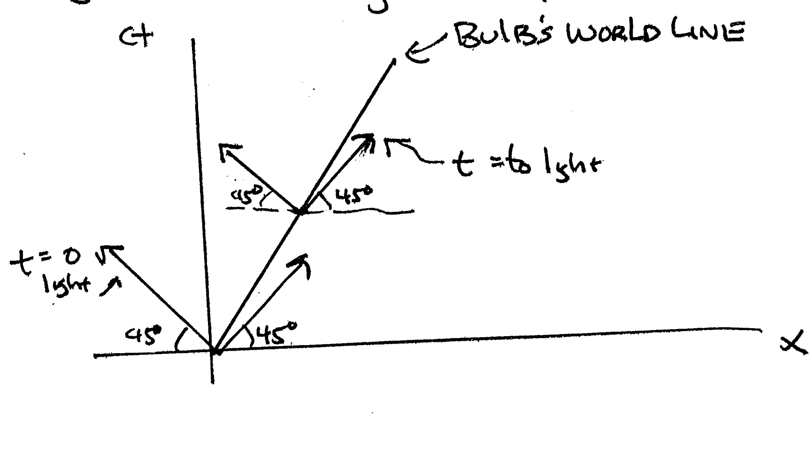
Slope = $\frac{c}{V}$

LIGHT CONE → WORLD LINE FOR BEAM OF LIGHT

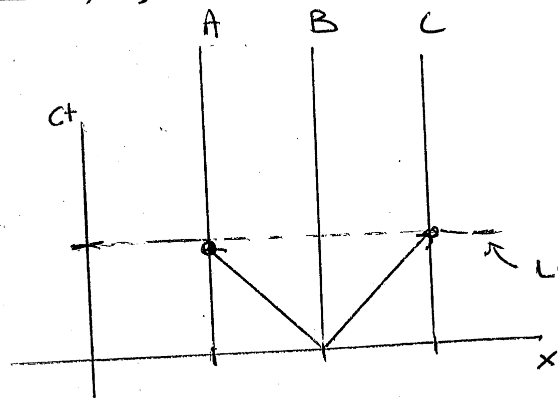
EX: Light Bulb at origin RADIATING IN All Directions.

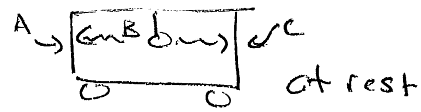


EX: LIGHT BULB MOVING WITH SPEED V.

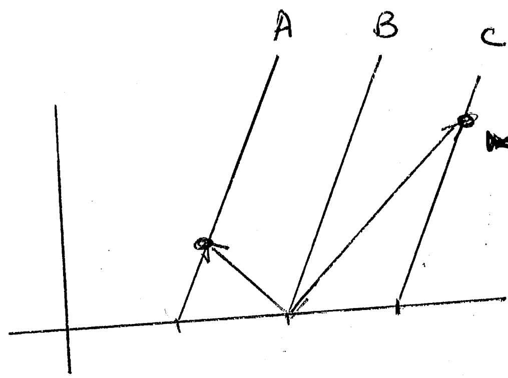


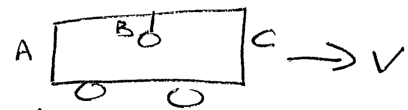
SIMULTANEITY





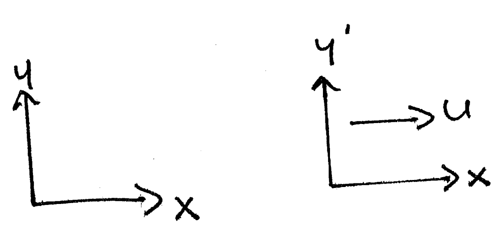
LIGHT HITS A & C AT SAME TIME.





LIGHT HITS C AFTER A

TWO FRAMES, S & S'

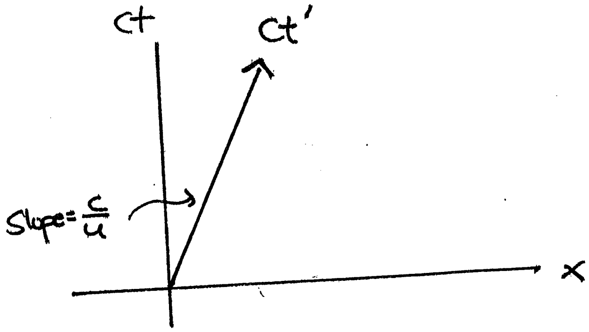


LORENTZ TRANSFORM:

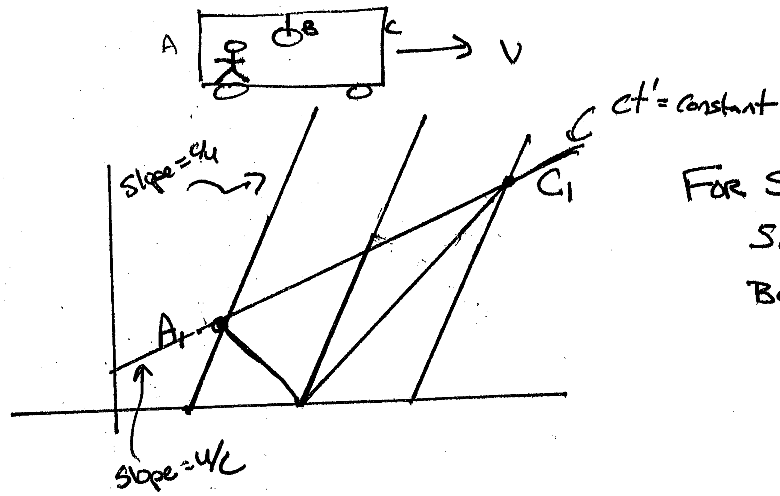
$$x' = \gamma(x - ut)$$
$$t' = \gamma\left(t - \frac{ux}{c^2}\right)$$
$$y' = y, \quad z' = z$$

ct' AXIS = LOCATION OF $x' = 0$. x' moving with speed u with respect to S

$\Rightarrow$



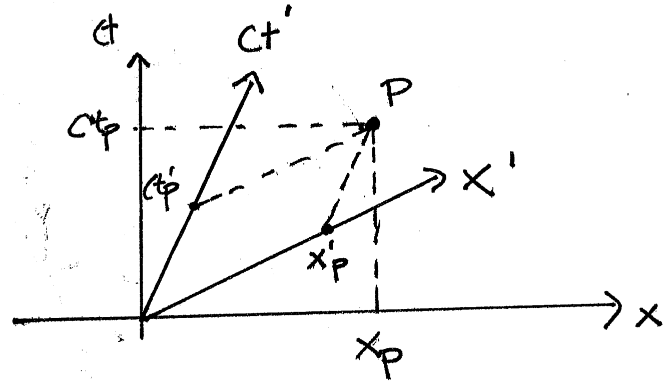
To find x', start WITH $ct' = $ CONSTANT $\Rightarrow$ SIMULTANEOUS EVENTS.



FOR SOMEONE ON RAILCAR, i.e. SOMEONE IN S' A_1 AND C_1 would Be simultaneous $\Rightarrow$ at $ct' =$ constant

X' AXIS $\Rightarrow ct' = 0 \Rightarrow t' = 0 \quad t' = \gamma\left(t - \frac{ux}{c^2}\right) \Rightarrow t = 0, x = 0 \Rightarrow$

x' AXIS = LINE parallel to one ABOVE passing through origin



P = POINT EVENT

OR JUST AN EVENT

BE CAREFUL! THE length SCALES Along x AND x' (As well As t & t') ARE NOT THE SAME.

↳ SEE BELOW

INVARIENT INTERVAL:

$$x' = \gamma(x - ut), \quad t' = \gamma\left(t - \frac{ux}{c^2}\right)$$

$$\Rightarrow [ct']^2 - x'^2 = c^2\gamma^2\left(t - \frac{ux}{c^2}\right)^2 - \gamma^2(x - ut)^2$$

$$= c^2\gamma^2\left(t^2 - \frac{2uxt}{c^2} + \frac{u^2x^2}{c^4}\right) - \gamma^2\left(x^2 - 2uxt + u^2t^2\right)$$

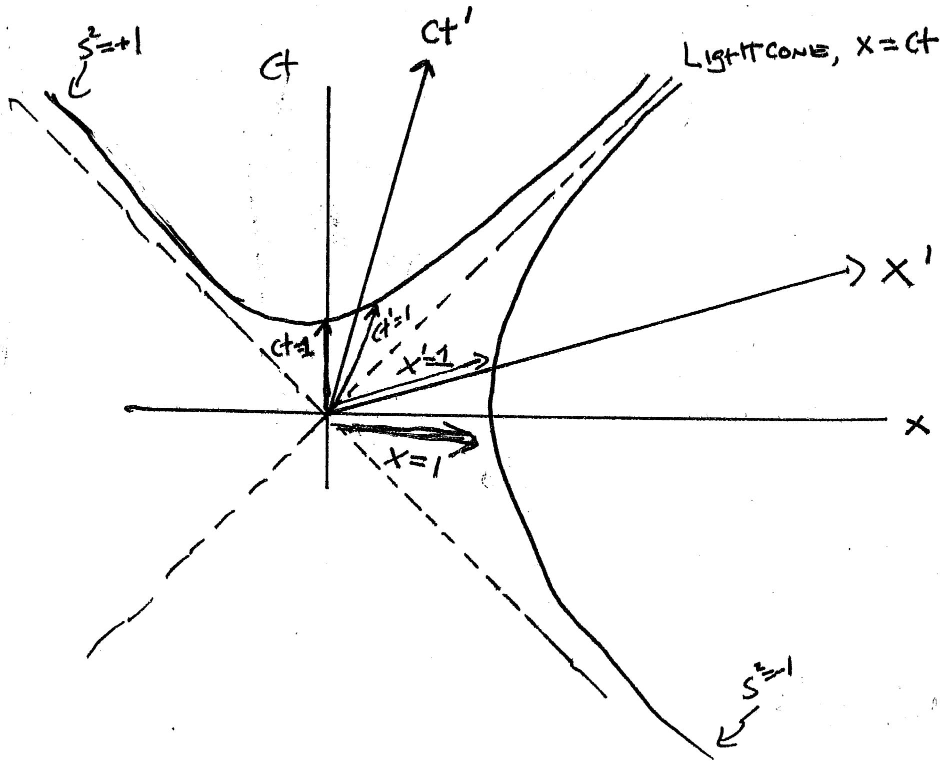
$$= \gamma^2 c^2 t^2 \left(1 - u^2/c^2\right) - \gamma^2 x^2 \left(1 - u^2/c^2\right) - \cancel{2\gamma^2 uxt} + \cancel{2\gamma^2 uxt}$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \Rightarrow \gamma^2 \left(1 - u^2/c^2\right) = 1$$

$\Rightarrow [ct']^2 - x'^2 = [ct]^2 - x^2 \leftarrow$ SAME FOR ALL ~~OBS~~ FRAMES OF REFRENCE!

$S^2 = [ct]^2 - x^2 =$ INVARIENT

PLOTS OF $S^2 =$ Constant ARE HYPERBOLAE $\Rightarrow$ MINKOWSKI DIAGRAM = HYPERBOLIC GEOMETRY.



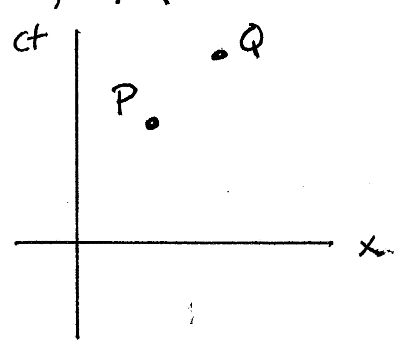
$S^2 = -1 \Rightarrow x^2 - [ct]^2 = 1$

AND $x'^2 - [ct']^2 = 1$

$S^2 = +1 \Rightarrow [ct]^2 - x^2 = 1$

AND $[ct']^2 - x'^2 = 1$

FOR TWO EVENTS, P, Q



$P = (x_1, ct_1)$

$Q = (x_2, ct_2)$

$\Delta x = x_2 - x_1$

$\Delta t = t_2 - t_1$

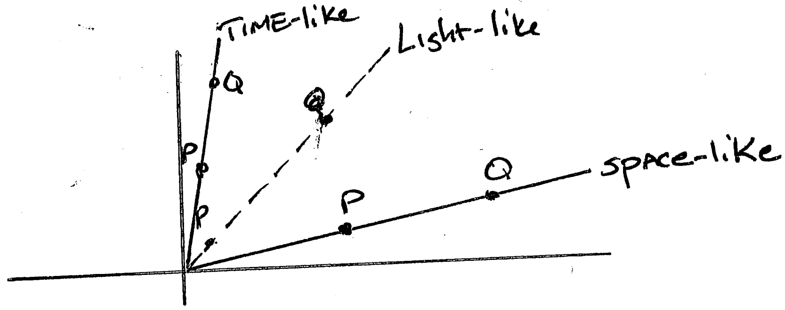
INVARIENT INTERVAL:

$$\boxed{\Delta S^2 = [c\Delta t]^2 - \Delta x^2}$$

If line connecting P, Q is at 45° $\Rightarrow \Delta S = 0 \rightarrow$ P & Q are "light-like" separated.

If $\Delta S^2 > 0$, P & Q are "time-like" separated.

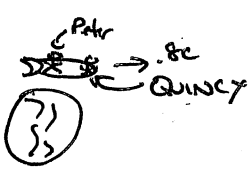
If $\Delta S^2 < 0$, P & Q are "space-like" separated



Time-like $\Rightarrow$ we can find a frame S' such that ct' passes through both P & Q
$\Rightarrow$ in S' occuring at same place but at different times.

Space-like $\Rightarrow$ frame S' such that x' passes through both P & Q
$\Rightarrow$ occuring simultaneously but at different locations.

Example: A spaceship is traveling by Earth with speed $u = .8c$ (relative to Earth).
At the instant the back of the spaceship ($x' = 0$) is even with Earth ($x = 0$)
Peter at the back and Quincy in the front of spaceship, both pop open a bottle of champaign. Sketch the spacetime diagram for events P & Q,
Find ΔS.



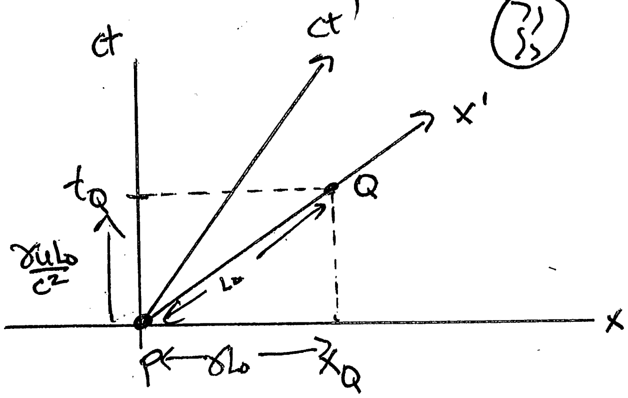
Let $t' = 0$ when bottles opened.

Peter: P at ($x' = 0, t' = 0$)

Quincy: Q at ($x' = L_0, t' = 0$) ← proper length of spaceship

$$\Delta S^2 = [c\Delta t']^2 - x'^2 = 0 - L_0^2$$

$= -L_0^2$ ← space-like separation



From diagram $x_P, t_P = 0, 0$

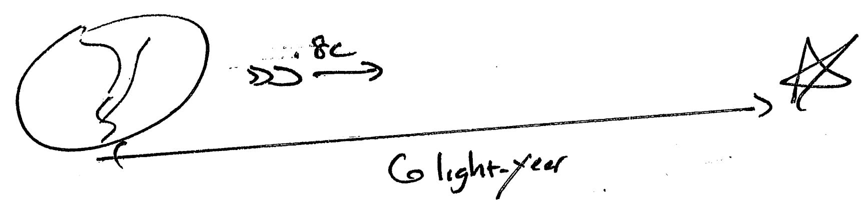
Lorentz: $x = \gamma(x' + ut') \Rightarrow x_Q = \gamma L_0$

$t = \gamma(t' + \frac{ux'}{c^2}) \Rightarrow t_Q = \frac{\gamma u L_0}{c^2}$

$$\Rightarrow \Delta S^2 = \left[c\frac{\gamma u L_0}{c^2}\right]^2 - [\gamma L_0]^2 = \frac{\gamma^2 u^2 L_0^2}{c^2} - \gamma^2 L_0^2 = \gamma^2 L_0^2\left(\frac{u^2}{c^2} - 1\right)$$

$$= -L_0^2$$

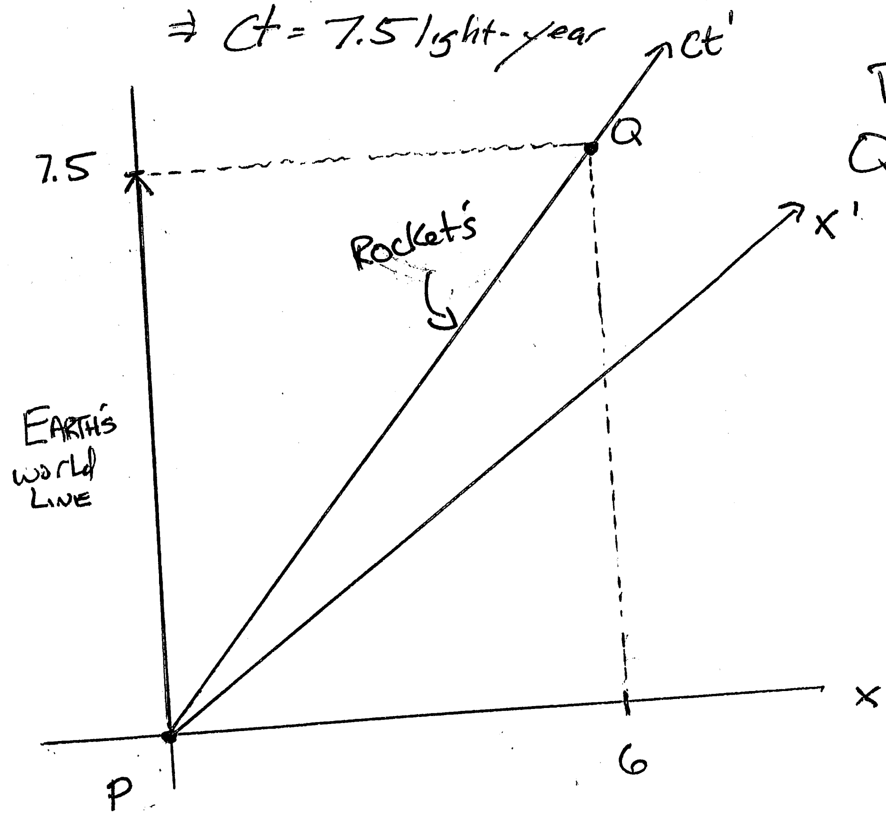
EX: A ROCKET LEAVES EARTH AT $t = t' = 0$ AND TRAVELS WITH SPEED $u = .8c$ (RELATIVE TO EARTH) TO A STAR 6 LIGHT-YEARS AWAY (AS MEASURED FROM EARTH). SKETCH THE ROCKET'S AND EARTH'S WORLD LINE. FIND Δs^2 FOR DEPARTURE & ARRIVAL EVENTS.



LET S = EARTH, S' = ROCKET

IN S, Rocket arrives at $x = 6$ light-year at $t = \frac{6}{.8c} = 7.5 \frac{\text{light-year}}{c}$

$\Rightarrow ct = 7.5$ light-year



P = Rocket Departs

Q = Rocket Arrives

P at $(0,0) = (x_1, t_1)$

Q at $(6, \frac{7.5}{c}) = (x_2, t_2)$

$\Rightarrow \Delta s^2 = [c \cdot \frac{7.5}{c}]^2 - 6^2$

$\Delta s^2 = 20.25$

$\Rightarrow$ P & Q timelike seperated

IN S', Rocket Always at $x' = 0$

$\Rightarrow$ Q on ct' LINE

INTERVAL FROM P to Q SAME IN S', but $x' = 0$

$\Rightarrow 20.25 = [c\Delta t']^2 - 0^2 \Rightarrow c\Delta t' = \sqrt{20.25} = 4.5$ light-year $\leftarrow$ SAME ANSWER AS TIME DILATION