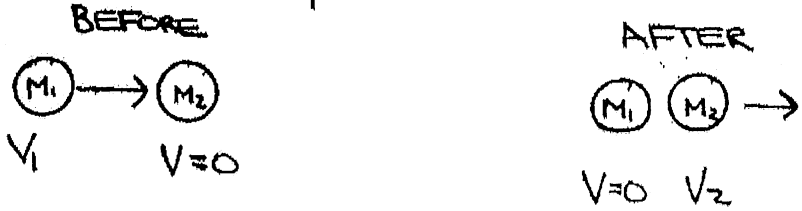
PHYS 262: RELATIVISTIC DYNAMICS AND ENERGY, CHAPTER 37

MOMENTUM – WE MUST HAVE CONSERVATION OF MOMENTUM IN ALL INERTIAL FRAMES. BUT THE ORIGINAL DEFINITION $\vec{p} = M\vec{v}$ CAUSES PROBLEMS.



$P_{TOTAL} = M_1V_1 + 0 = M_1V_1$ $\qquad$ $P_{TOTAL} = 0 + M_2V_2 = M_2V_2$

CONSERVATION OF MOMENTUM $\Rightarrow M_1V_1 = M_2V_2 \Rightarrow V_2 = \dfrac{M_1V_1}{M_2}$

LET $M_1 = 3M_2$, $V_1 = .5c \Rightarrow V_2 = \dfrac{3M_2(.5c)}{M_2} = 1.5c \rightarrow$ IMPOSSIBLE!

EINSTEIN FOUND THAT THE PROPER FORM FOR MOMENTUM IS

$$\boxed{\vec{p} = \gamma M_0 \vec{v}} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

THIS IS OFTEN INTERPRETED AS SAYING THAT AS VELOCITY INCREASES SO DOES MASS.

$M = \gamma M_0$. $\quad M_0$ = REST MASS

NOTICE THAT AS $V \to c$, $M \to \infty \Rightarrow$ IT TAKES AN INFINITE AMOUNT OF FORCE TO ACCELERATE AN OBJECT. THIS IS WHY IT'S IMPOSSIBLE FOR AN OBJECT WITH MASS TO REACH THE SPEED OF LIGHT.

EXAMPLE: FIND V_2 FOR $M_1 = 3M_2$, $V_1 = .5c$ USING RELATIVISTIC MOMENTUM.

ASSUME REST MASSES M_1 AND M_2 $\Rightarrow$

BEFORE: $P_{TOTAL} = \gamma_1 M_1 V_1 = \frac{1}{\sqrt{1-.5^2}}(3M_2)(.5c) = \frac{1.5M_2 c}{\sqrt{1-.5^2}}$

AFTER: $P_{TOTAL} = \gamma_2 M_2 V_2 = \frac{1}{\sqrt{1-V_2^2/c^2}} M_2 V_2$

$$\Rightarrow \frac{1.5\cancel{M_2}c}{\sqrt{1-.5^2}} = \frac{\cancel{M_2}V_2}{\sqrt{1-V_2^2/c^2}} \Rightarrow \left(\frac{1.5c}{\sqrt{1-.5^2}}\right)^2 = \left(\frac{V_2}{\sqrt{1-V_2^2/c^2}}\right)^2 \Rightarrow \frac{2.25c^2}{1-.5^2} = \frac{V_2^2}{1-V_2^2/c^2}$$

$$\Rightarrow 3c^2 = \frac{V_2^2}{1-V_2^2/c^2} \Rightarrow 3c^2(1-V_2^2/c^2) = V_2^2 \Rightarrow 3c^2 - 3V_2^2 = V_2^2$$

$$\Rightarrow 4V_2^2 = 3c^2 \Rightarrow V_2 = \sqrt{\tfrac{3}{4}}\,c = .866c$$

FORCE FIND THE FORCE IS A LITTLE TRICKY. WE'LL RESTRICT OURSELVES TO THE CASE WHERE $\vec{F}$ AND $\vec{V}$ ARE PARALLEL.

LET $\vec{V} = V\hat{\imath} \Rightarrow P_x = \gamma M_0 V$.

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow F_x = \frac{d}{dt}(\gamma M_0 V) = \frac{d}{dt}\left(\frac{M_0 V}{\sqrt{1-V^2/c^2}}\right) =$$

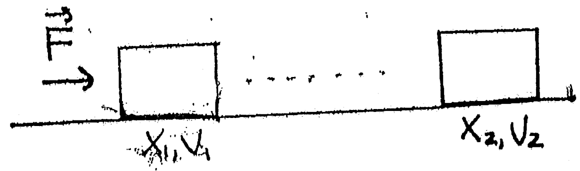
$$M_0 V\left(-\frac{1}{2}\right)(1-V^2/c^2)^{-3/2}\left(\frac{-2V}{c^2}\right)\left(\frac{dV}{dt}\right) + \frac{1}{\sqrt{1-V^2/c^2}} M_0 \frac{dV}{dt} = \frac{M_0 V^2 a}{c^2(1-V^2/c^2)^{3/2}} + \frac{M_0 a}{(1-V^2/c^2)^{1/2}}$$

$$\Rightarrow F_x = \frac{M_0 a}{(1-V^2/C^2)^{3/2}}\left[\frac{V^2}{C^2} + 1 - \frac{V^2}{C^2}\right] = \frac{M_0 a}{(1-V^2/C^2)^{3/2}}$$

$$\Rightarrow \boxed{F_x = \gamma^3 M_0 a} \qquad \vec{F} \text{ AND } \vec{V} \text{ PARALLEL}$$

KINETIC ENERGY – USE THE WORK-ENERGY THEOREM

$$\Delta K = W = \int \vec{F} \cdot d\vec{\ell}$$



ASSUME STRAIGHT LINE MOTION IN X-DIRECTION

$$\Rightarrow \Delta K = \int_{X_1}^{X_2} F_x \, dx = \int_{X_1}^{X_2} \gamma^3 M_0 a \, dx = \int_{X_1}^{X_2} \frac{M_0 a}{(1-V^2/C^2)^{3/2}} dx$$

$$a = \frac{dV}{dt} \Rightarrow a\,dx = \frac{dV}{dt} dx = dV \frac{dx}{dt} = dV\,V$$ (WITH CORRESPONDING CHANGE OF VARIABLE)

$$\Rightarrow \Delta K = \int_{V_1}^{V_2} \frac{M_0 V dV}{(1-V^2/C^2)^{3/2}}$$ SUBSTITUTE $u = 1 - V^2/C^2 \Rightarrow du = -\frac{2V}{C^2} dV$

$$\Rightarrow \Delta K = \int_{u_1}^{u_2} -\frac{M_0 C^2}{2} \frac{du}{u^{3/2}} = -\frac{M_0 C^2}{2} \int_{u_1}^{u_2} \frac{du}{u^{3/2}} = M_0 C^2 u^{-1/2} \Big|_{u_1}^{u_2}$$

$$u_1 = 1 - \frac{V_1^2}{C^2}, \; u_2 = 1 - \frac{V_2^2}{C^2} \Rightarrow \Delta K = \frac{M_0 C^2}{\sqrt{1-V_2^2/C^2}} - \frac{M_0 C^2}{\sqrt{1-V_1^2/C^2}}$$

FOR THE CASE $V_1 = 0,\ V_2 = V \Rightarrow \Delta K = \frac{M_o c^2}{\sqrt{1-v^2/c^2}} - M_o c^2$

$\Delta K = K_2 - K_1$. $V_1 = 0 \Rightarrow K_1 = 0$, LET $K_2 = K$

$\Rightarrow K = \frac{M_o c^2}{\sqrt{1-v^2/c^2}} - M_o c^2 \Rightarrow \boxed{K = (\gamma - 1) M_o c^2}$

TO SEE CLASSIC FORM OF KINETIC ENERGY:

$\gamma = (1 - V^2/c^2)^{-1/2} = 1 - \frac{1}{2}(-V^2/c^2) + \ldots \approx 1 + \frac{1}{2} V^2/c^2$ FOR $V \ll C$

$\Rightarrow K = (1 + \frac{1}{2}\frac{V^2}{c^2} - 1) M_o c^2 = \frac{1}{2}\frac{V^2}{c^2}(M_o c^2) = \frac{1}{2} M_o V^2$

$K = \gamma M_o c^2 - \underbrace{M_o c^2}$

$\quad \hookrightarrow V = 0$ ENERGY = REST ENERGY

$\Rightarrow \gamma M_o c^2 = K + M_o c^2 \qquad \Rightarrow \boxed{E = \gamma M_o c^2}$

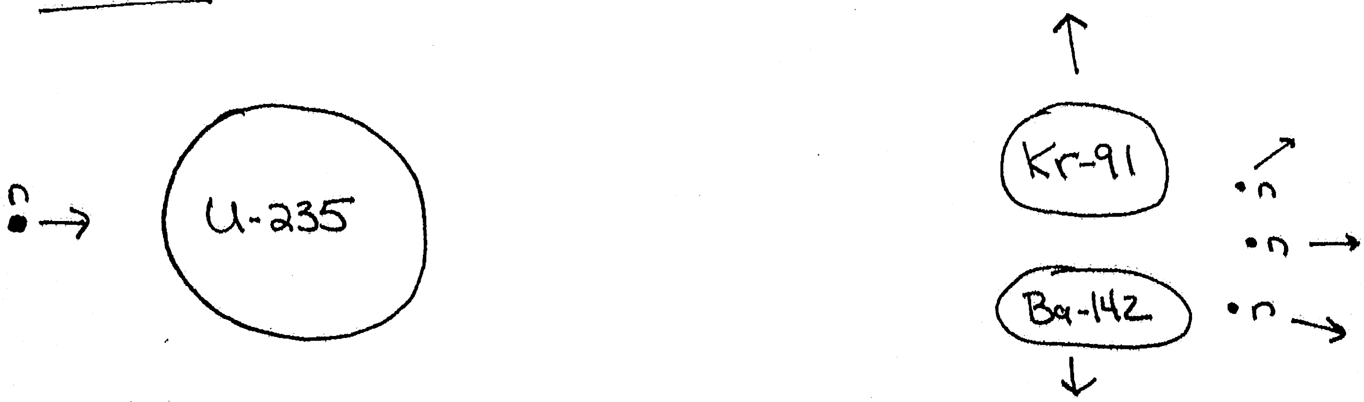
KINETIC POTENTIAL

THIS LOOKS A LITTLE DIFFERENT FROM THE FAMOUS $E = Mc^2$ BECAUSE $M = \gamma M_o$.

$E = \gamma M_o c^2 = K + M_o c^2$ TELLS US THAT MASS IS ENERGY. c^2 IS THE CONSTANT CONVERSION FACTOR. AT REST $E = M_o c^2$. UNIT: $J = Kg\,(m/s)^2 = Kg\,m^2/s^2 = N \cdot m$

CONSERVATION OF ENERGY TELLS US THAT ENERGY CHANGES FORM $\Rightarrow$ MASS MAY BE CONVERTED INTO KINETIC OR POTENTIAL ENERGY (OR VICE-VERSA).

<u>FISSION</u> - SPLITTING OF AN ATOM



BEFORE

AFTER

n = NEUTRON. $M_n = 1.67493 \times 10^{-27}$ kg

U-235 = URANIUM 235 = 143 NEUTRONS
92 PROTONS

$M_u = 3.90437 \times 10^{-25}$ Kg

Kr-91 = KRYPTON 91 = 55 NEUTRONS
36 PROTONS

$M_{Kr} = 1.51036 \times 10^{-25}$ Kg

Ba-142 = BARIUM 142 = 86 NEUTRONS
56 PROTONS

$M_{Ba} = 2.35741 \times 10^{-25}$ Kg

TOTAL MASS AFTER FISSION:

$$M_{Kr} + M_{Ba} + 3M_n = 3.91802 \times 10^{-25} \text{ Kg}$$

TOTAL MASS BEFORE FISSION:

$$M_u + M_n = 3.921112 \times 10^{-25} \text{ Kg}$$

ENERGY CREATED: $E = \Delta M C^2 = .0031 \times 10^{-25} \text{ Kg} \, (3 \times 10^8 \text{ m/s})^2 = 2.79 \times 10^{-11} \text{ J}$

THIS EXTRA ENERGY IS MOSTLY IN THE KINETIC ENERGY of the KRYPTON AND BARIUM ATOM.

$$K_2 = K_{Kr} + K_{Ba} + K_{n_1} + K_{n_2} + K_{n_3}$$

$$K_1 = K_n + K_U$$

$\Rightarrow K_2 > K_1 \rightarrow$ THIS IS <u>NOT</u> AN ELASTIC "COLLISION".

<u>FUSION</u> – COMBINING OF LIGHTER NUCLEI TO MAKE HEAVIER ELEMENTS

THE SIMPLEST FUSION EVENT IS TWO HYDROGEN ATOMS FUSING TO MAKE HELIUM.

BEFORE

AFTER

D = DEUTERIUM = 1 PROTON
1 NEUTRON

$M_D = 3.343 \times 10^{-27}$ Kg

He = Helium = 2 PROTONS
2 NEUTRONS

$M_{He} = 6.644 \times 10^{-27}$ Kg

ENERGY CREATED: $(M_{He} - 2M_D)c^2 = (.042 \times 10^{-27}\,\text{Kg})(3\times10^8\,\text{m/s})^2 = 3.78 \times 10^{-12}\,\text{J}$

STARS CREATE ENERGY BY FUSING HYDROGEN INTO HELIUM; HOWEVER, IT'S MUCH MORE COMPLICATED THAN ABOVE. IN THE "PROTON-PROTON CHAIN" 6 PROTONS ARE CONVERTED INTO A HELIUM ATOM AND TWO FREE PROTONS, WHICH GO ON AND CAUSE OTHER FUSION EVENTS.