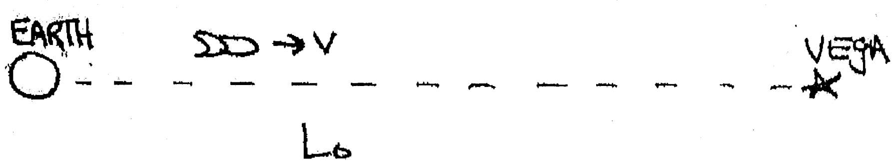
PHYS 262: SPECIAL RELATIVITY II, CHAPTER 37

LORENTZ CONTRACTION – TWO OBSERVERS MOVING RELATIVE TO EACH OTHER MEASURE DIFFERENT LENGTHS.

L_0 = PROPER LENGTH. LENGTH IN THE INERTIAL FRAME IN WHICH THE OBJECT IS NOT MOVING.

EXAMPLE: A SPACESHIP LEAVES EARTH AND TRAVELS TO THE STAR VEGA. WHO MEASURES THE PROPER LENGTH FOR THE EARTH–VEGA DISTANCE?



DISTANCE FROM EARTH TO VEGA NOT CHANGING TO SOMEONE ON EARTH, SO THEY MEASURE THE PROPER LENGTH.

– WHO MEASURES THE PROPER TIME?

THE EVENT IS THE SPACESHIP GOING TO VEGA. A WATCH ON THE SPACESHIP IS NOT MOVING RELATIVE TO SOMEONE ON THE SPACESHIP, SO THEY MEASURE PROPER TIME.

TO FIND THE AMOUNT OF LENGTH CONTRACTION, WE USE THE FACT THAT EVERYBODY AGREES ON THE SPACESHIP'S SPEED, V.

EARTH: $V = \frac{L_0}{\Delta t}$

SPACESHIP: $V = \frac{L}{\Delta t_0}$

$$\Rightarrow \frac{L_0}{\Delta t} = \frac{L}{\Delta t_0} \Rightarrow L = L_0 \frac{\Delta t_0}{\Delta t} = L_0 \frac{\Delta t_0}{\gamma \Delta t_0}$$

$$\Rightarrow \boxed{L = \frac{L_0}{\gamma}}$$

AS V INCREASES, L DECREASES

EXAMPLE WHAT DISTANCE FROM EARTH TO JUPITER DOES AN ELECTRON WITH $V=.79c$ MEASURE? WE ON EARTH MEASURE A DISTANCE 6.28×10^{11}m.

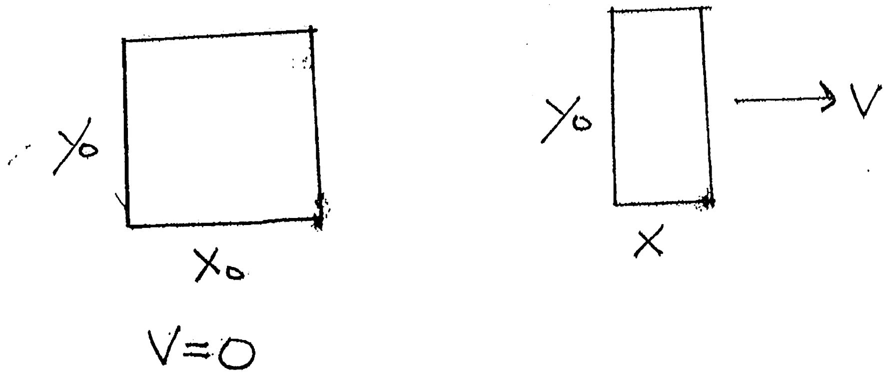
FOR REASONS SIMILAR TO BEFORE, EARTH MEASURES THE PROPER LENGTH. $L_o = 6.28\times10^{11}$m

$$L=\frac{L_o}{\gamma}\quad \gamma=\frac{1}{\sqrt{1-v^2/c^2}}=1.631 \Rightarrow L=\frac{6.28\times10^{11}\text{m}}{1.631}=3.85\times10^{11}\text{m}$$

LORENTZ CONTRACTION (AKA LENGTH CONTRACTION) IS THE FLIPSIDE TO TIME DILATION, THE ONLY WAY THE ELECTRON CAN MEASURE (i.e., TRAVEL) A SHORTER TIME IS IF IT TRAVELS A SHORTER DISTANCE.

NOTE: AS $V\to C$, $\gamma\to\infty \Rightarrow L\to 0$. IT IS IMPOSSIBLE FOR ATOMS TO BE SQUEEZED DOWN TO NOTHING $\Rightarrow$ ANYTHING WITH MASS CANNOT GO AT THE SPEED OF LIGHT. WHICH MEANS THERE IS AN ULTIMATE SPEED LIMIT.

ALSO, LENGTH CONTRACTION OCCURS ONLY ALONG THE DIRECTION OF THE RELATIVE MOTION.

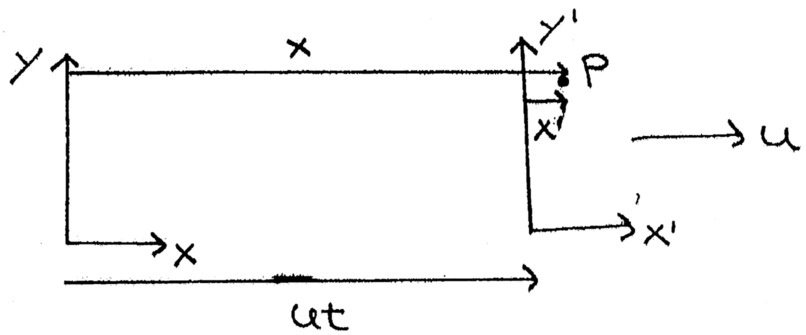


CONTRACTED ALONG X BUT NOT Y.

LORENTZ TRANSFORMATION - A CHANGE OF CO-ORDINATES IS MORE FUN IN SPECIAL RELATIVITY.

FOR TWO FRAMES S AND S' WITH CO-ORDINATES (x, y, z, t) AND (x', y', z', t') RESPECTIVELY, THE LORENTZ TRANSFORM CHANGES FROM (x, y, z, t) to (x', y', z', t')

GALILEAN TRANSFORM: NON RELATIVITY.



$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

LORENTZ TRANSFORM

IMAGINE THAT THE POINT P IS NOT MOVING IN THE S' FRAME.

x' = LENGTH TO P IN S' FRAME $\Rightarrow x'$ = PROPER LENGTH.

IN S, WE MEASURE THE CONTRACTED LENGTH x'/γ $\Rightarrow$

$$x = ut + \frac{x'}{\gamma} \Rightarrow x' = \gamma(x - ut).$$

NO LENGTH CONTRACTION ALONG y OR z $\Rightarrow y' = y$, $z' = z$.

FROM S' POINT OF VIEW, S IS MOVING TO LEFT WITH VELOCITY $-u$.

$\Rightarrow$ THEY MEASURE CONTRACTED LENGTH FOR x, i.e., x/γ AND A DISTANCE BETWEEN S AND S' OF ut'.

$$\Rightarrow x' = \frac{x}{\gamma} - ut'$$ (SAME VALUE OF γ BECAUSE $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$ HAS $\pm u$ SYMMETRY).

$$x' = \gamma(x-ut) \Rightarrow \gamma(x-ut) = \frac{x}{\gamma} - ut'$$

$$\Rightarrow ut' = \frac{x}{\gamma} - \gamma(x-ut) = x\left(\frac{1}{\gamma} - \gamma\right) + \gamma ut$$

$$\Rightarrow ut' = x\gamma \underbrace{\left(\frac{1}{\gamma^2} - 1\right)}_{1-u^2/c^2-1 \,=\, -u^2/c^2} + \gamma ut \quad \Rightarrow ut' = \gamma\left(x\left(\frac{-u^2}{c^2}\right) + ut\right)$$

$$\Rightarrow ut' = \gamma u\left(t - \frac{xu}{c^2}\right) \Rightarrow t' = \gamma\left(t - \frac{xu}{c^2}\right)$$

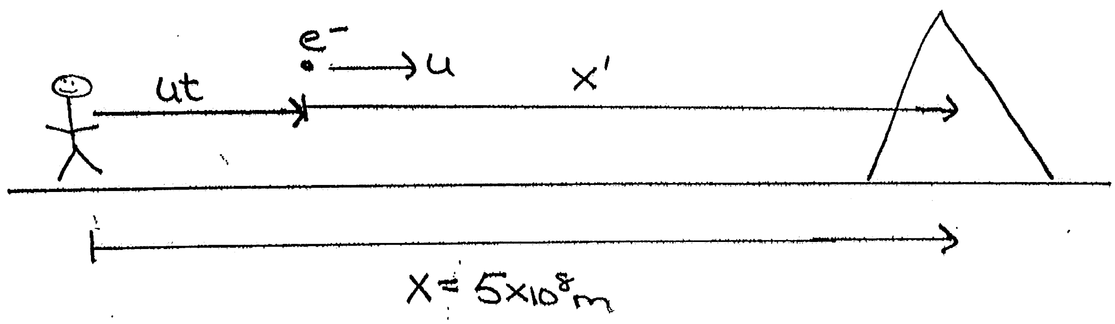
$$\boxed{\begin{aligned} x' &= \gamma(x-ut) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{xu}{c^2}\right) \end{aligned}}$$

LORENTZ TRANSFORMATION

TO GET INVERSE TRANSFORM, WE CAN SOLVE ALGEBRAICALLY FOR x, y, z, t. <u>OR</u> WE CAN REMEMBER THAT IN S', S MOVES TO LEFT WITH VELOCITY $-u$ $\Rightarrow$

$$x = \gamma(x' + ut')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{x'u}{c^2}\right)$$

EXAMPLE: AN ELECTRON MOVING WITH A SPEED $u = .79c$ IS HEADED TOWARDS A MOUNTAIN. YOU ARE A STATIONARY DISTANCE OF 5×10^8m* AWAY FROM THE MOUNTAIN. WHAT DISTANCE DOES THE ELECTRON MEASURE FOR VARIOUS TIMES ACCORDING TO THE GALILEAN AND LORENTZ TRANSFORM. (*AS MEASURED BY YOU.)



GALILEAN: $x' = x - ut = 5\times10^8 m - .79(3\times10^8 m/s)t$, $t' = t$

t	x	t'	x'
0	5×10^8m	0	5×10^8m
1s	5×10^8m	1s	2.6×10^8m
2s	5×10^8m	2s	$.26\times10^8$m
2.11s	5×10^8m	2.11s	0

LORENTZ: $x' = \gamma(x - ut)$, $t' = \gamma\left(t - \frac{xu}{c^2}\right)$ $\gamma = \frac{1}{\sqrt{1-(.79)^2}} = 1.631$

t	x	t'	x'
0	5×10^8m	-2.15s	8.16×10^8m
1s	5×10^8m	$-.52$s	4.28×10^8m
2s	5×10^8m	1.11s	$.424\times10^8$m
2.11s	5×10^8m	1.29s	0

IN YOUR FRAME, WHEN $t=0$, THE ELECTRON IS DIRECTLY ABOVE YOU.

IN ELECTRON'S FRAME IT WILL BE 2.15s BEFORE IT IS DIRECTLY ABOVE YOU. WHAT IS SIMULTANEOUS IN YOUR FRAME (MOUNTAIN AND ELECTRON BOTH 5×10^8m AWAY) IS <u>NOT</u> SIMULTANEOUS IN ELECTRON'S INERTIAL FRAME.

TIME DILATION IS A LITTLE TRICKY TO SEE WITH LORENTZ TRANSFORM.

THE EVENT IS THAT THE ELECTRON GOES FROM YOU TO THE MOUNTAIN.

ELECTRON MEASURES PROPER TIME $\Rightarrow \Delta t_0 = 1.29\text{s}$

YOU MEASURE DILATED TIME $\Rightarrow \Delta t = 2.11\text{s}$

$$\frac{2.11\text{s}}{1.29\text{s}} = 1.63 = \gamma$$

LENGTH CONTRACTION: ELECTRON GOES WITH $u = .79c$ FOR 1.29s

$$\Rightarrow L = .79(3\times10^8 \text{m/s})(1.29\text{s}) = 3.06\times10^8\text{m}$$

$$L_0 = 5\times10^8\text{m} \quad \Rightarrow \frac{L_0}{L} = 1.63 = \gamma$$

VELOCITY ADDITION - THE LORENTZ TRANSFORM ALLOWS US TO FIND THE SPEEDS IN S'.

$V' =$ SPEED IN FRAME S'.

$$V_x' = \frac{dx'}{dt'} \qquad \begin{aligned} dx' &= \gamma(dx - u\,dt) \\ dt' &= \gamma\left(dt - \frac{u\,dx}{c^2}\right) \end{aligned} \qquad u = \text{CONSTANT}$$

$$\Rightarrow V_x' = \frac{\gamma(dx - u\,dt)}{\gamma\left(dt - \frac{u\,dx}{c^2}\right)} = \frac{dt\left(\frac{dx}{dt} - u\right)}{dt\left(1 - \frac{u}{c^2}\frac{dx}{dt}\right)} = \frac{V_x - u}{1 - \frac{uV_x}{c^2}}$$

$$\boxed{V_x' = \frac{V_x - u}{1 - \frac{uV_x}{c^2}}} \quad \Rightarrow \quad \boxed{V_x = \frac{V_x' + u}{1 + \frac{uV_x'}{c^2}}}$$

$$V_y' = \frac{dy'}{dt'} \quad . \quad dy' = dy \Rightarrow V_y' = \frac{dy}{\gamma\left(dt - \frac{u\,dx}{c^2}\right)} = \frac{dy}{\gamma\,dt\left(1 - \frac{u}{c^2}\frac{dx}{dt}\right)}$$

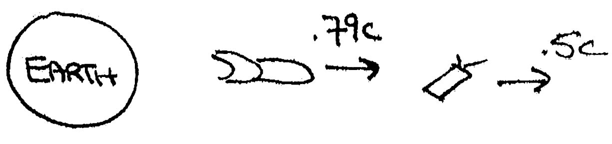
$$\Rightarrow \boxed{V_y' = \frac{V_y}{\gamma\left(1 - \frac{uV_x}{c^2}\right)}} \Rightarrow \boxed{V_y = \frac{V_y'}{\gamma\left(1 + \frac{uV_x'}{c^2}\right)}}$$

LIKEWISE: $\boxed{V_z' = \frac{V_z}{\gamma\left(1 - \frac{uV_x}{c^2}\right)}}$ $\boxed{V_z = \frac{V_z'}{\gamma\left(1 + \frac{uV_x'}{c^2}\right)}}$

NOTICE WHEN $V_x = c \Rightarrow V_x' = \frac{c-u}{1 - \frac{uc}{c^2}} = \frac{c-u}{1-\frac{u}{c}} = \frac{c-u}{\frac{c-u}{c}} = c$

THIS ADDITION DOESN'T ALLOW VELOCITIES LARGER THAN C.

EXAMPLE: A SPACESHIP TRAVELLING AT .79c (RELATIVE TO EARTH) LAUNCHES A SPACEPROBE WITH A SPEED THE SPACESHIP MEASURES TO BE .5c. WHAT SPEED DO WE MEASURE ON EARTH?



$u = .79c, \; V_x' = .5c$

$$V_x = \frac{V_x' + u}{1 + \frac{uV_x'}{c^2}} = \frac{.5c + .79c}{1 + (.79)(.5)} = \frac{1.29c}{1.395} = .925c$$