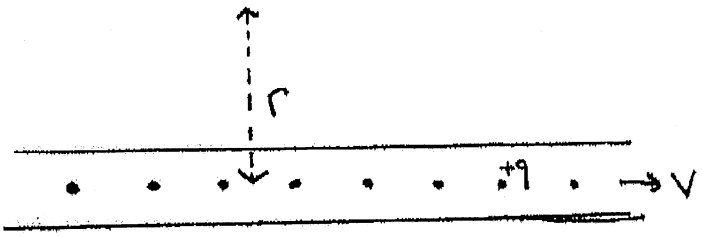
Phys 262 Special Relativity I, Chapter 37

Problem with Electromagnetism!

Current create magnetic fields:



$B = \frac{\mu_0 I}{2\pi r}$. $I = \frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = \lambda v$

λ: charge per length

$\Rightarrow B = \frac{\mu_0 \lambda v}{2\pi r}$.

If we "run along" with the charges (at same velocity), they would be stationary relative to us. Stationary charges create electric fields. Using Gauss's law, the electric field of a wire is

$$E = \frac{2k\lambda}{r} .$$

Notice that $E \neq B(v=0)$ → something is wrong! Electric fields should become (transform) into magnetic fields depending on whether the charges are moving relative to an observer.

In 1905, Einstein published his solution to this problem. He called it his theory of special relativity.

Relativity - motion of objects relative to each other.

Special - objects moving with constant velocity.

Postulates of Special Relativity

1. Equivalence of physical laws → The laws of physics are the same in all inertial frames of reference.
2. Constancy of speed of light → The speed of light in a vacuum (3×10^8 m/s) is the same in all inertial frames of reference, independent of the motion of the source or receiver.

Inertial frame of reference – Any area of the universe where Newton's laws are obeyed (an object with zero net force has no acceleration.).

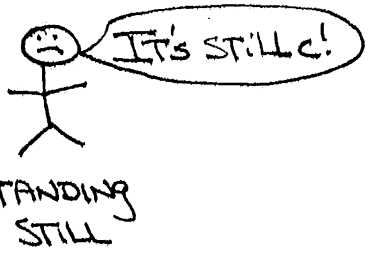
Examples: Any train, boat, or rocketship moving with constant speed.

Technically the Earth is not an inertial frame because of the centripetal acceleration from the Earth's rotation about its axis and around the sun. These forces are very small; however so we are approximately inertial.

Equivalence of physical laws ⇒ all laws. Not just Newton's law, but conservation laws, E&M, thermodynamics, etc.

Example: The electric field created by a line of charge must be $E = \frac{2k\lambda}{r}$ in all frames in which the charges are at rest. It does not matter, that the charges are moving in other inertial frames.

CONSTANCY OF SPEED OF LIGHT → CRAZY, COUNTER-INTUITIVE!

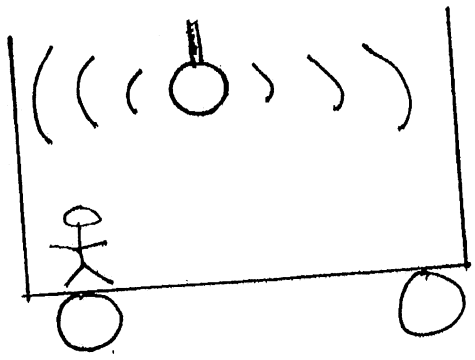


STANDING STILL PERSON DOES NOT MEASURE A SPEED FOR THE LIGHT TO BE 1.5c.

HOW??? – WE ACCEPT IT AS A POSTULATE (AN ASSUMPTION). THE CONSEQUENCES OF THIS ASSUMPTION CAN BE EXPERIMENTALLY TESTED. IF THE CONSEQUENCES ARE VERIFIED THEN OUR ASSUMPTION (NO MATTER HOW FREAKY) MUST ALSO BE TRUE.

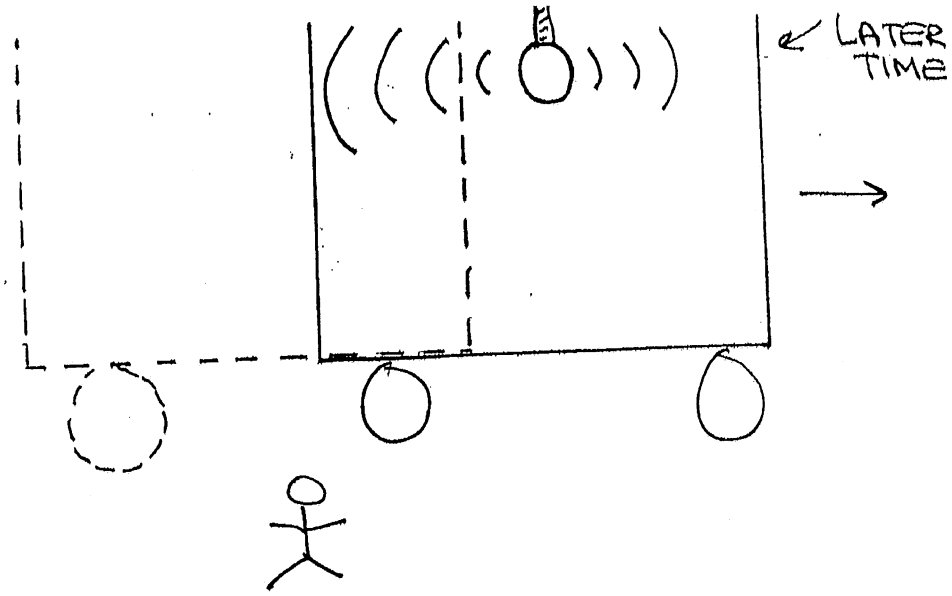
SIMULTANEITY – SIMULTANEOUS EVENTS ARE TWO OCCURANCES THAT HAPPEN AT EXACTLY THE SAME TIME.

IN RELATIVITY, EVENTS WHICH ARE SIMULTANEOUS IN ONE INERTIAL FRAME MAY NOT BE SIMULTANEOUS IN ANOTHER.



PERSON INSIDE OF MOVING TRAIN CAR TURNS ON A LIGHT IN THE CENTER OF THE CAR. SHE "SEES" LIGHT REACH BOTH ENDS AT THE SAME TIME. THEY ARE SIMULTANEOUS.

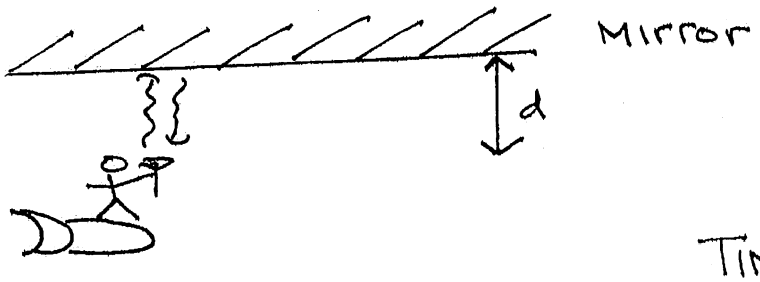
A PERSON ON THE GROUND MEASURES THE SAME SPEED FOR THE LIGHT. IF TRAIN IS MOVING TO THE RIGHT, THE LIGHT HITS THE LEFT SIDE BEFORE THE RIGHT SIDE.



LEFT SIDE "CATCHES UP" TO THE LIGHT WHILE RIGHT SIDES MOVES FARTHER AWAY.

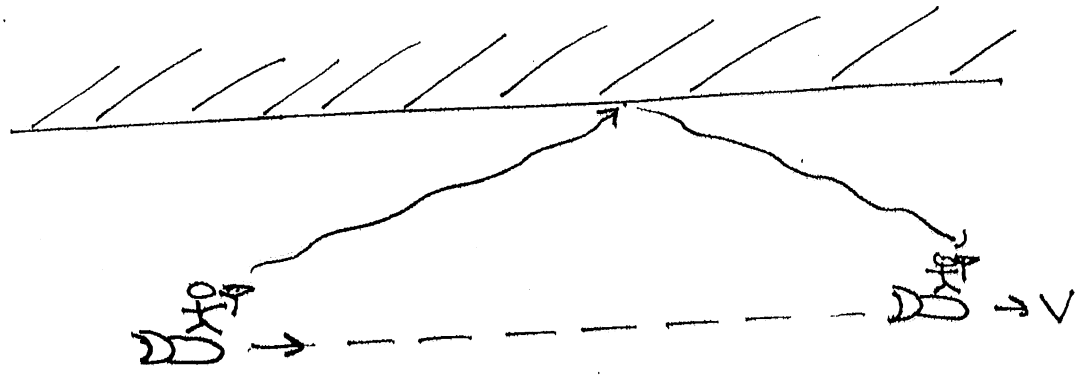
WHEN SOMETHING OCCURS DEPENDS ON WHERE THE OBSERVER IS LOCATED. THIS IS WHY WE SAY EINSTEIN LINKED SPACE AND TIME. WE REFER TO SPACETIME. TO LOCATE AN EVENT, WE NEED TO GIVE ITS (x, y, z, t) CO-ORDINATES. AND t WILL NOT BE THE SAME FOR EVERYONE.

TIME DILATION – CLOCKS (OF ALL TYPES) RUN SLOWER IN MOVING INERTIAL FRAMES.



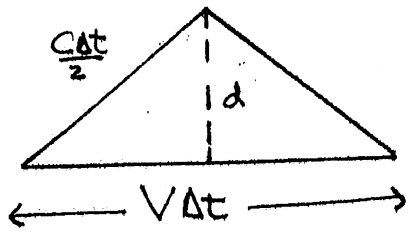
ROCKETSHIP AT REST

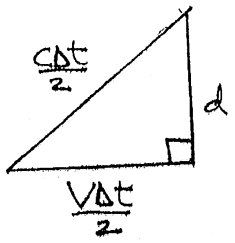
TIME FOR LIGHT TO TRAVEL TO MIRROR AND BACK IS $\Delta t_0 = \frac{2d}{c}$



ROCKET MOVES WITH SPEED V

IF ENTIRE TRIP TAKES A TIME Δt, THE SHIP MOVES A DISTANCE $V\Delta t$, LIGHT TRAVELS A DISTANCE $C\Delta t$.





$$\left(\frac{V\Delta t}{2}\right)^2 + d^2 = \left(\frac{C\Delta t}{2}\right)^2$$

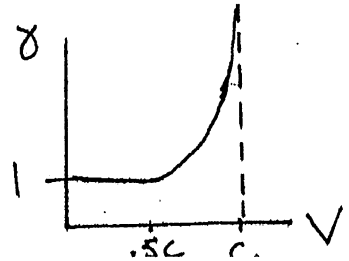
$$\frac{V^2}{4}\Delta t^2 + d^2 = \frac{C^2}{4}\Delta t^2 \Rightarrow \Delta t^2\left(\frac{C^2-V^2}{4}\right) = d^2 \Rightarrow \Delta t^2 C^2 (1 - V^2/C^2) = 4d^2$$

$$\Rightarrow \Delta t^2 (1 - V^2/C^2) = \frac{4d^2}{C^2} \Rightarrow \Delta t \sqrt{1 - V^2/C^2} = \frac{2d}{C} = \Delta t_0$$

$$\Rightarrow \boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - V^2/C^2}} = \gamma \Delta t_0} \qquad \boxed{\gamma = \frac{1}{\sqrt{1 - V^2/C^2}}}$$

LORENTZ FACTOR

Δt_0 = PROPER TIME. THE TIME TAKEN BY AN EVENT IN THE INERTIAL FRAME IN WHICH POSITION IS NOT CHANGING.



FOR "LOW" SPEEDS ($V < .6c$), $\gamma \approx 1$. RELATIVISTIC EFFECTS ARE EXTREMELY DIFFICULT TO OBSERVE.

In 1971, two atomic clocks were put on comercial airline planes and flown around the world (one to the east the other west). When they were re-united with a 3rd clock that had remained at the U.S. naval observatory. The airline clocks had recorded elapsed times that were a few billionths of a second slower! Just as relativity predicts.

Example: An electron is moving with a speed of .79c. How long is the time measured by the electron to reach Jupiter from earth, a distance of 6.28×10^{11}m (as measured from earth)?

The event here is the electron going from earth to Jupiter. If we put a watch on the electron, people on earth see the watch moving $\Rightarrow$ they measure the dilated time, Δt. Someone riding on the electron, sees a stationary watch $\Rightarrow$ they measure the proper time, Δt_0.

The distance 6.28×10^{11}m is measured from earth $\Rightarrow d = v\Delta t$

$$6.28\times10^{11}\text{m} = .79\,(3\times10^{8}\text{m/s})\,\Delta t \Rightarrow \Delta t = 2650\text{s} = 44\text{min}$$

$$\Delta t_0 = \frac{\Delta t}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-\frac{(.79c)^2}{c^2}}} = \frac{1}{\sqrt{1-.79^2}} = 1.631$$

$$\Rightarrow \Delta t_0 = \frac{2650\text{s}}{1.631} = 1625\text{s} = 27\text{min}$$

Note: Time dilation occurs for all clocks including biological clocks. So someone riding on electron has lived 27min. Someone on earth has lived 44min. The electron person is younger!

— WHILE THE ELECTRON IS TRAVELLING TO JUPITER A FIRE BREAKS OUT AT TV1. THE FIREFIGHTERS MEASURE A TIME OF 16MIN TO PUT IT OUT. HOW LONG IS THE TIME AS MEASURED ON THE ELECTRON?

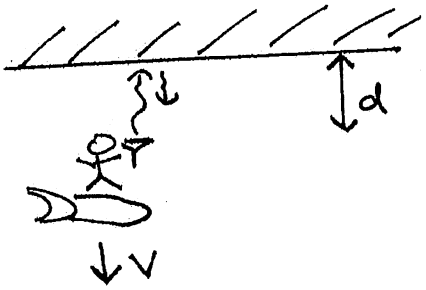
NOW THE EVENT IS THE FIRE BEING PUT OUT. IF WE PUT A WATCH ON THE FIRE. WE ON EARTH SEE A STATIONARY WATCH SO WE MEASURE THE PROPER TIME $\Rightarrow \Delta t_0 = 16\text{MIN}$.

SOMEONE ON THE ELECTRON SEES THE FIRE AND THEREFORE THE WATCH MOVING SO THEY MEASURE THE DILATED TIME Δt.

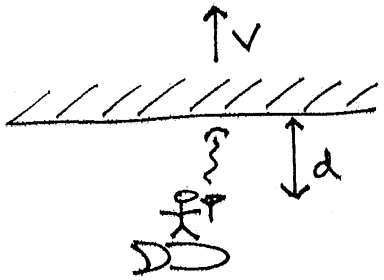
$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t = (1.631)(16\text{MIN}) = 26\text{MIN}$.

REMEMBER THERE IS NO "ABSOLUTE" FRAME WHICH ALWAYS MEASURES THE PROPER TIME.

THERE IS NO TIME DILATION FOR MOTION P...



LIGHT TAKES t_1 TO REACH MIRROR $\Rightarrow t_1 = d/c$.
TO RETURN IT TAKES t_2. DURING THE TRIP UP, THE SHIP MOVED BY $Vt_1 = Vd/c$. DURING THE TRIP DOWN, THE SHIP MOVES BY Vt_2. SO THE LIGHT HAS TO GO A DISTANCE $d + \frac{Vd}{c} + Vt_2$.
$\Rightarrow ct_2 = d + \frac{Vd}{c} + Vt_2 \Rightarrow t_2 = \frac{d}{c-V} + \frac{dV}{c(c-V)}$
$\Rightarrow \Delta t = t_1 + t_2 = \frac{d}{c} + \frac{d}{c-V} + \frac{dV}{c(c-V)} = \frac{2d}{c-V}$



DURING TRIP UP, THE MIRROR MOVES Vt_3, SO LIGHT GOES A DISTANCE $d + Vt_3$
$\Rightarrow ct_3 = d + Vt_3 \Rightarrow t_3 = \frac{d}{c-V}$
TO RETURN LIGHT HAS TO TRAVEL AN ADDITIONAL $d + Vt_3$ SINCE SPACESHIP IS STATIONARY
$\Rightarrow \Delta t = 2t_3 = \frac{2d}{c-V}$ → TIMES FOR EACH TRIP IS THE SAME.