PHYS 262 : INTERFERENCE CONTINUED, CHAPTER 35

INTENSITY OF INTERFERING OF WAVES.

$I = S_{AV}$, $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ (POYNTING VECTOR)

FROM $E = Bc$, WE GET $S = \epsilon_0 c E^2$.

WITH INTERFERENCE, WE HAVE TWO OR MORE FIELDS AT A POINT.

PLANE WAVE INTENSITY

PLANE WAVE: $\vec{E} = \vec{E}_0 \cos(kz - \omega t)$.

LET'S ASSUME WE HAVE TWO COHERENT SOURCES OF EQUAL MAGNITUDE THAT ARE LINEARLY POLARIZED ALONG THE X-AXIS $\Rightarrow$

$$\vec{E}_1 = \hat{\imath}\, E_0 \cos(kz_1 - \omega t), \quad \vec{E}_2 = \hat{\imath}\, E_0 \cos(kz_2 - \omega t)$$

WHERE $k(z_1 - z_2) = \phi$ HAS AN UNCHANGING VALUE WITH TIME.

THE TOTAL ELECTRIC FIELD AT A POINT IS:

$$\vec{E}_p = \hat{\imath}\, E_0 \left(\cos(kz_1 - \omega t) + \cos(kz_2 - \omega t) \right)$$

$$\Rightarrow E_p^2 = E_0 \left[\cos(kz_1 - \omega t) + \cos(kz_2 - \omega t) \right]^2$$

TO SIMPLIFY, WE USE THE TRIG IDENTITY:

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\Rightarrow \cos(kz_1-\omega t)+\cos(kz_2-\omega t) = 2\cos\left(\frac{kz_1-\omega t+kz_2-\omega t}{2}\right)\cos\left(\frac{kz_1-\omega t-kz_2+\omega t}{2}\right)$$

$$= 2\cos\left(\frac{k(z_1+z_2)}{2}-\omega t\right)\cos\left(\frac{k(z_1-z_2)}{2}\right)$$

$$= 2\cos\left(\frac{k(z_1+z_2)}{2}-\omega t\right)\cos(\phi/2)$$

$$\therefore\ E_p^2 = E_o^2(4)\cos^2\left(\frac{k(z_1+z_2)}{2}-\omega t\right)\cos^2\left(\frac{\phi}{2}\right)$$

$$S=\epsilon_o c E_t^2 \Rightarrow S = 4\epsilon_o c\cos^2(\phi/2)\cos^2\left(\frac{k(z_1+z_2)}{2}-\omega t\right)E_o^2$$

$I = S_{AV}$ WHERE WE NEED TO TAKE A TIME AVERAGE.

THE AVERAGE OF $\cos^2\left(\frac{k(z_1+z_2)}{2}-\omega t\right) = 1/2$

$$\Rightarrow \boxed{I = 2\epsilon_o c E_o^2\cos^2(\phi/2)}$$

THE MAXIMUM INTENSITY OCCURS WHERE $\cos^2(\phi/2)=1 \Rightarrow \phi = 0$

$$\Rightarrow I_o = 2\epsilon_o c E_o^2(1) = 2\epsilon_o c E_o^2$$

$$\Rightarrow \boxed{I = I_o\cos^2(\phi/2)}$$

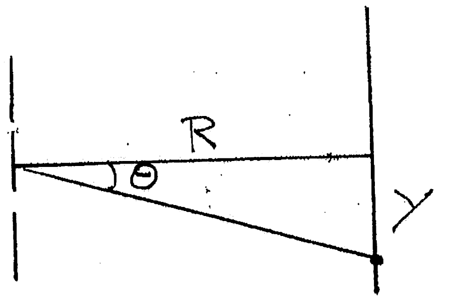
THIS EQUATION FOR INTENSITY IS TRUE IN GENERAL FOR ALL EM WAVES; HOWEVER, ϕ MUST BE MODIFIED AS $\boxed{\phi = k(r_1-r_2)}$

FOR YOUNG'S DOUBLE SLIT EXPERIMENT:

$$r_1 - r_2 = d\sin\theta \Rightarrow \frac{\phi}{K} = d\sin\theta \Rightarrow \phi = Kd\sin\theta$$

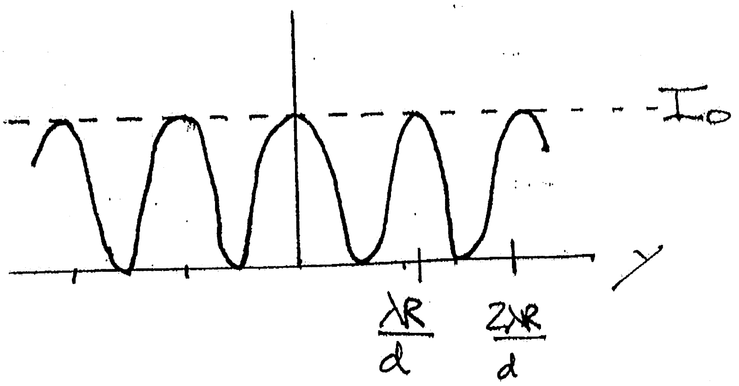
$$\left(K = \frac{2\pi}{\lambda}\right) \quad \Rightarrow \phi = \frac{2\pi d\sin\theta}{\lambda}$$

$$\Rightarrow I = I_o \cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$



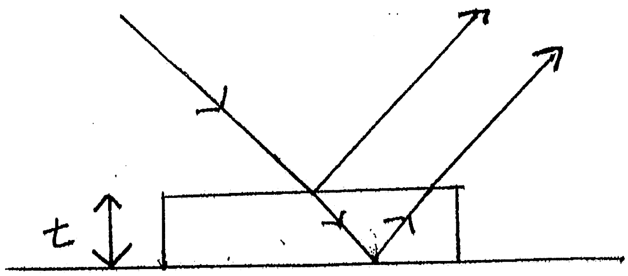
$y = R\tan\theta$

FOR θ SMALL, $\tan\theta \approx \sin\theta \Rightarrow I = I_o \cos^2\left(\frac{\pi d}{\lambda R} y\right)$



$\hookrightarrow y = \frac{\lambda R}{d} \Rightarrow \lambda = \frac{yd}{R} = \frac{R\tan\theta\, d}{R} = d\tan\theta \approx d\sin\theta$

<u>THIN FILMS</u> - THIN LAYERS OF OIL ON WATER CREATE SHIFTING BANDS OF COLOR. THIS IS CAUSED BY INTERFERENCE BETWEEN THE REFLECTED LIGHT AND THE REFRACTED THEN REFLECTED LIGHT.

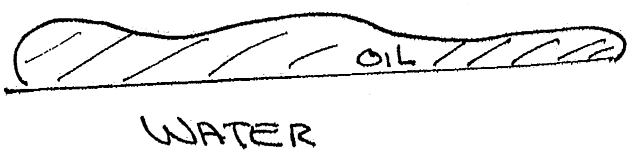


WHEN THE TWO BEAMS ARE BROUGHT TOGETHER (BY YOUR EYE USUALLY), THERE IS INTERFERENCE.

IF THE FILM IS VERY THIN, THEN $r_1 - r_2$ (THE PATH LENGTH DIFFERENCE IS APPROXIMATELY $2t$ = TWICE THE FILM'S THICKNESS. THIS IS A GOOD APPROXIMATION TO MAKE BECAUSE THIN FILMS OFTEN HAVE THICKNESSES ON THE ORDER OF THE LIGHT'S WAVELENGTH ($\sim 10^{-7}$ m FOR VISIBILE LIGHT)

$\Rightarrow$ CONSTRUCTIVE INTERFERENCE OCCURS WHEN $2t = m\lambda$
DESTRUCTIVE INTERFERENCE OCCURS WHEN $2t = (m + \frac{1}{2})\lambda$.

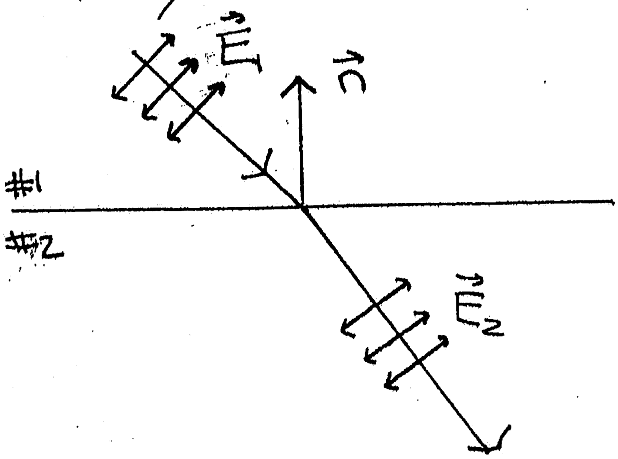
WE SEE THE RAINBOW FROM THIN FILMS BECAUSE DIFFERENT WAVELENGTHS FROM WHITE LIGHT HAVE DIFFERENT CONSTRUCTIVE INTERFERENCE THICKNESSES.



THICKNESS CHANGES OVER OIL SLICK'S SURFACE.

IF THERE IS AN AIR BUBBLE IN THE FILM; HOWEVER, THE CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE ARE SWITCHED. THIS IS BECAUSE THERE IS SOMETIMES A 180° PHASE SHIFT IN $\vec{E}$ UPON REFLECTION.

THIS PHASE SHIFT CAN BE DERIVED FROM MAXWELL'S EQUATION. THEY GIVE US BOUNDARY CONDITIONS.



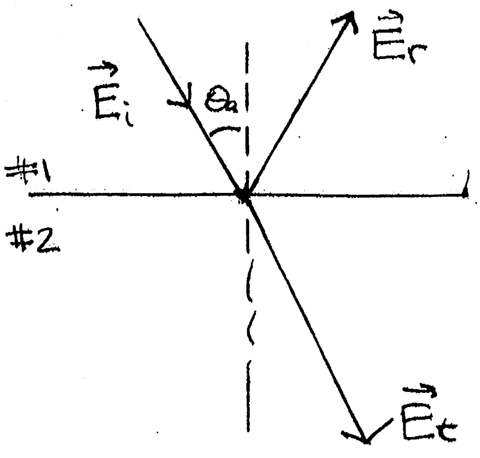
THE BOUNDARY CONDITIONS ON $\vec{E}$ ARE

$$\vec{n} \cdot (\vec{E}_2 - \vec{E}_1) = \sigma / \epsilon_0 \quad (\sigma = \text{SURFACE CHARGE DENSITY})$$

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$\vec{n} \cdot \vec{E}$ IS THE COMPONENT OF $\vec{E}$ PARALLEL TO THE NORMAL.
$\vec{n} \times \vec{E}$ IS THE COMPONENT PERPENDICULAR TO THE NORMAL.

FOR PLANE WAVES INCIDENT ON A CHARGE-FREE ($\sigma = 0$) SURFACE, THE BOUNDARY CONDITIONS TELL US THAT:



$\vec{E}_i$ = INCIDENT, $\vec{E}_r$ = REFLECTED, $\vec{E}_t$ = TRANSMITTED

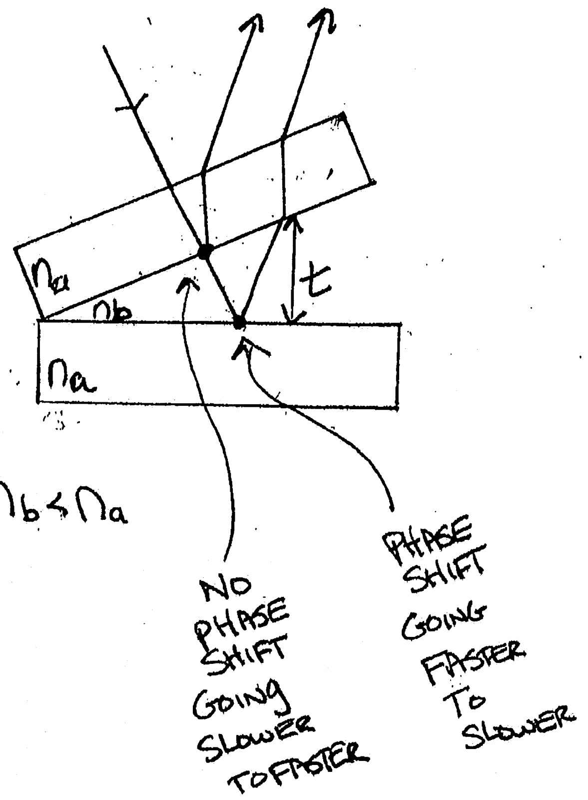
FOR $\theta_a \to 0$ (AS IS THE CASE WITH THIN FILMS)

$$E_r = \left(\frac{n_1 - n_2}{n_1 + n_2}\right) E_i$$

[IF YOU'RE CURIOUS $E_t = \frac{2n_1}{n_1 + n_2} E_i$]

WHEN $n_1 - n_2 < 0$ THEN E_r IS 180° OUT OF PHASE WITH E_i

THIS OCCURS WHEN $n_1 < n_2$, i.e., GOING FROM FASTER TO SLOWER.



SO THE TWO BEAMS ARE ALREADY 180° OUT OF PHASE WHEN THEY INTERFERE $\Rightarrow \vec{E}_p = \hat{\imath}(E_0)\left(\cos(kz_1 - \omega t) - \cos(kz_2 - \omega t)\right)$

DESTRUCTIVE INTERFERENCE OCCURS WHEN

$$\cos(kz_1 - \omega t) = \cos(kz_2 - \omega t) \Rightarrow kz_1 - \omega t = kz_2 - \omega t + 2\pi m$$

$$\Rightarrow k(z_1 - z_2) = 2\pi m \Rightarrow z_1 - z_2 = m\lambda$$

$\Rightarrow$ DESTRUCTIVE INTERFERENCE : $2t = m\lambda$

CONSTRUCTIVE INTERFERENCE : $2t = (m + \frac{1}{2})\lambda$