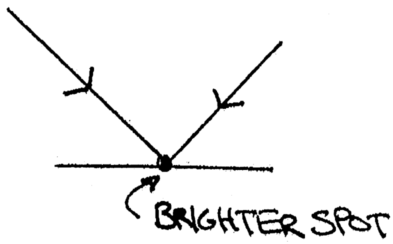
PHYS 2102 INTERFERENCE, CHAPTER 35

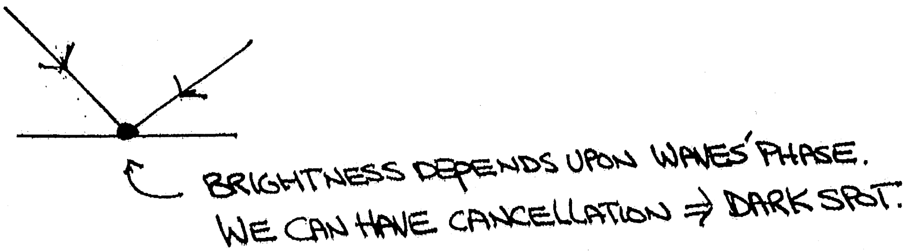
PHYSICAL OPTICS - LIGHT BEHAVIORS WHICH CANNOT BE EXPLAINED BY THE RAY THEORY (GEOMETRIC OPTICS).

INTERFERENCE - OCCURS WHEN TWO OR MORE WAVES OVERLAP AT THE SAME POINT.

GEOMETRIC OPTICS:



PHYSICAL OPTICS:



WE USE SUPERPOSITION TO FIND RESULTS OF INTERFERENCE.

SUPERPOSITION - THE RESULTANT DISPLACEMENT OF TWO OR MORE WAVES AT ANY POINT AND AT ANY INSTANT OF TIME IS FOUND BY ADDING THE DISPLACEMENTS THAT WOULD BE PRODUCED BY THE INDIVIDUAL WAVES IF EACH WERE PRESENT ALONE.

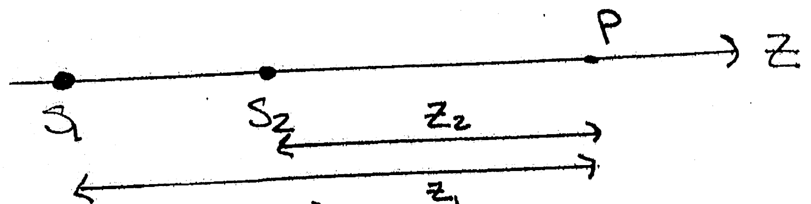
HERE DISPLACEMENT IS USED IN A GENERAL SENSE. FOR WATER WAVES IT WOULD BE THE WATER'S HEIGHT. FOR EM WAVES, DISPLACEMENT WOULD BE THE VALUES OF ELECTRIC AND MAGNETIC FIELD COMPONENTS.

PLANE WAVE INTERFERENCE – ASSUME THAT WE HAVE TWO PLANE WAVE SOURCES, S_1 AND S_2, WHICH ARE MONOCHROMATIC.

MONOCHROMATIC – LIGHT OF A SINGLE FREQUENCY. (LASERS CREATE MONOCHROMATIC LIGHT.)

LET $f_1 = S_1$'S FREQUENCY AND $f_2 = S_2$'S FREQUENCY.

FOR A PLANE WAVE : $\vec{E} = \vec{E}_0 \cos(Kz - \omega t)$. $\omega = 2\pi f$, $K = \frac{2\pi}{\lambda}$, $f\lambda = c$



AT POINT P: $\vec{E}_1 = \vec{E}_{01} \cos(K_1 z_1 - \omega_1 t)$, $\vec{E}_2 = \vec{E}_{02} \cos(K_2 z_2 - \omega_2 t)$

SUPERPOSITION TELLS US THAT AT THE POINT P, THE NET FIELD $\vec{E}_P$ IS

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

FOR SIMPLICITY, ASSUME LINEAR POLARIZATION ALONG X-AXIS $\Rightarrow$

$$\vec{E}_{01} = E_{01}\hat{\imath}, \quad \vec{E}_{02} = E_{02}\hat{\imath}$$

AND $f_1 = f_2 \Rightarrow K_1 = K_2$ AND $\omega_1 = \omega_2$

$$\Rightarrow \vec{E}_P = \hat{\imath}\left(E_{01}\cos(Kz_1 - \omega t) + E_{02}\cos(Kz_2 - \omega t)\right)$$

$-1 \le \cos\theta \le 1 \Rightarrow |E_P|_{MAX} = |E_{01} + E_{02}| \rightarrow$ CONSTRUCTIVE INTERFERENCE

$|E_P|_{MIN} = |E_{01} - E_{02}| \rightarrow$ DESTRUCTIVE INTERFERENCE

CONSTRUCTIVE INTERFERENCE OCCURS WHEN

$$\cos(kz_1-\omega t)=\cos(kz_2-\omega t)$$

$$\Rightarrow kz_1-\omega t = kz_2-\omega t+2\pi m \quad m=0,\pm1,\pm2,\ldots \text{ (ANY INTEGER)}$$

$$\Rightarrow k(z_1-z_2)=2\pi m \quad \Rightarrow \frac{2\pi}{\lambda}(z_1-z_2)=2\pi m$$

$$\Rightarrow \boxed{z_1-z_2=m\lambda}$$

DESTRUCTIVE INTERFERENCE OCCURS WHEN

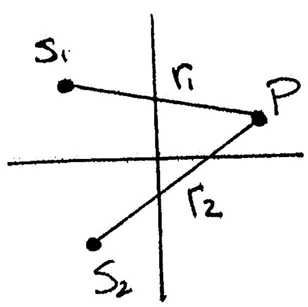
$$\cos(kz_1-\omega t)=-\cos(kz_2-\omega t)=\cos(kz_2-\omega t+(m+\tfrac{1}{2})2\pi)$$

$$\Rightarrow kz_1-\omega t = kz_2-\omega t+(m+\tfrac{1}{2})2\pi \quad m=0,\pm1,\pm2,\ldots$$

$$\Rightarrow \boxed{z_1-z_2=(m+\tfrac{1}{2})\lambda}$$

COHERENT LIGHT – THE CONDITIONS $f_1=f_2$ AND SAME DIRECTIONS FOR $\vec{E}_{01}$ AND $\vec{E}_{02}$ CREATE COHERENT LIGHT, LIGHT CREATED BY TWO MONOCHROMATIC, IDENTICAL SOURCES WHERE THERE IS A SINGLE DIFFERENCE IN PHASE.

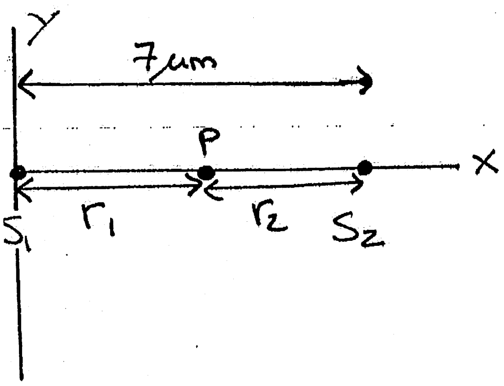
GENERAL COHERENT SOURCES – FOR NON PLANE WAVES THE SAME ARGUMENTS APPLY; HOWEVER, WHAT'S IMPORTANT IS THE DIFFERENCE IN PATH LENGTH.



$$\boxed{\begin{array}{l}\text{CONSTRUCTIVE INTF.} \Rightarrow r_1-r_2=m\lambda \\ \text{DESTRUCTIVE INTF.} \Rightarrow r_1-r_2=(m+\tfrac{1}{2})\lambda\end{array}}$$

EXAMPLE: TWO COHERENT LIGHT SOURCES ARE SEPERATED BY $7\mu m$. WHERE ARE THE POINTS OF CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE FOR GREEN LIGHT ($\lambda = 525nm$) ALONG THE LINE CONNECTING THE TWO SOURCES.

WE CAN SET UP CO-ORDINATE SYSTEM HOWEVER WE LIKE. SO PUT S_1 AT ORIGIN AND S_2 $7\mu m$ AWAY ALONG X.



$$r_1 = x$$

$$r_2 = 7\mu m - x$$

$$\Rightarrow r_1 - r_2 = 2x - 7\mu m$$

CONSTRUCTIVE $\Rightarrow 2x - 7\mu m = m(525nm) = m(.525\mu m)$

$$\Rightarrow x = m\left(\frac{.525\mu m}{2}\right) + \tfrac{7}{2}\mu m = m(.2625\mu m) + 3.5\mu m$$

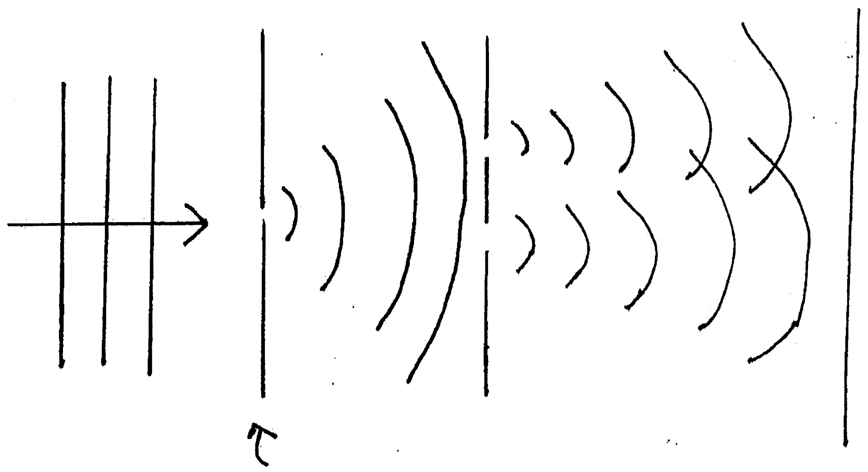
m	x
-3	2.7µm
-2	2.98µm
-1	3.24µm
0	3.5µm
1	3.76µm
2	4.03µm

DESTRUCTIVE $\Rightarrow 2x - 7\mu m = (m+\frac{1}{2})(.525\mu m) \Rightarrow$

$$x = (m+\tfrac{1}{2})(.2625\mu m) + 3.5\mu m$$

m	x
-3	2.8 μm
-2	3.1 μm
-1	3.4 μm
0	3.6 μm
1	3.9 μm

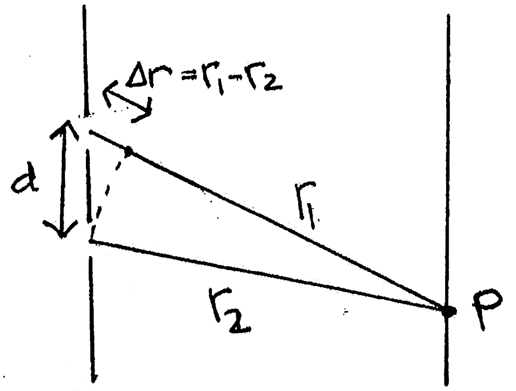
~~YOUNG'S~~ DOUBLE SLIT EXPERIMENT — FAMOUS EXPERIMENT WHICH "PROVED" LIGHT IS A WAVE. IN 1801, THOMAS YOUNG SENT MONO-CHROMATIC LIGHT THROUGH THE FOLLOWING SET OF SLITS.



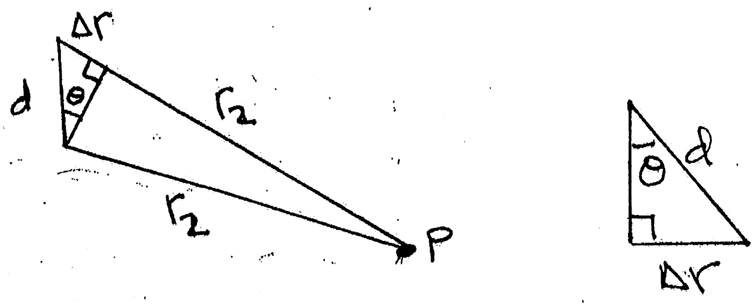
LIGHT FROM THE TWO SLITS INTERFERE MAKING STRIPS OF LIGHT AND DARK (p. 1344)

MAKES A SMALL "POINT" SOURCE

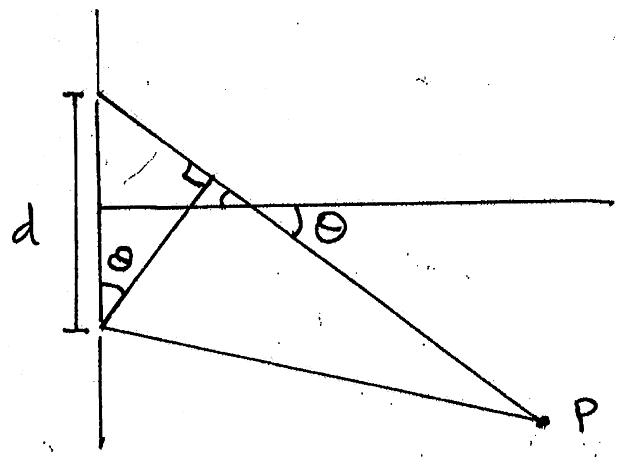
TO GET NICE EQUATIONS, WE NEED TO MAKE SOME SIMPLIFICATIONS.



IF d IS VERY SMALL (IF THE SLITS ARE CLOSE TOGETHER) THEN THE ARC CONNECTING r_1 AND r_2 IS APPROXIMATELY A STRAIGHT LINE PERPENDICULAR TO r_2

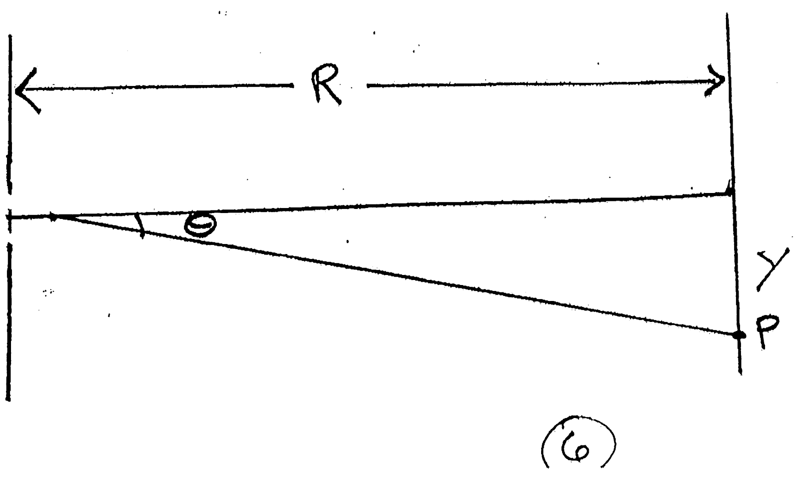


$\Delta r = d \sin\theta$



θ IS THE ANGLE FROM WHERE THE CENTER OF SLITS INTERCEPTS r TO THE POINT P.

IF THE FINAL SCREEN'S DISTANCE, R, IS VERY LARGE, THE DISTANCE FROM THE CENTER TO THE INTERCEPTION POINT BECOMES NEGLIGIBLE.



$\tan\theta \approx y/R$

CONSTRUCTIVE INTERFERENCE OCCURS WHEN $r_1 - r_2 = m\lambda$

$r_1 - r_2 = \Delta r = d\sin\theta$

$\Rightarrow \boxed{d\sin\theta = m\lambda}$ $\quad m = 0, \pm 1, \pm 2,$

DESTRUCTIVE INTERFERENCE OCCURS WHEN $r_1 - r_2 = (m + \tfrac{1}{2})\lambda$

$\Rightarrow \boxed{d\sin\theta = (m + \tfrac{1}{2})\lambda}$

THE CENTER OF THE STRIPS OCCUR AT $y = R\tan\theta$

FOR VERY SMALL ANGLES, $\tan\theta \approx \sin\theta \Rightarrow y = R\sin\theta \Rightarrow$

$y = \dfrac{Rm\lambda}{d}$ FOR CONSTRUCTIVE, $\quad y = \dfrac{R(m + \tfrac{1}{2})\lambda}{d}$ FOR DESTRUCTIVE