PHYS 262 : HW#8 40.1, 40.11, 40.43, 40.44, 40.46, 40.48

40.1 $M = .2\text{ Kg}, L = 1.5\text{m}$. FIND GROUND STATE

$E_n = \frac{\pi^2\hbar^2}{2ML^2} n^2$ FOR PARTICLE IN A BOX

$$\Rightarrow E_1 = \frac{\pi^2(1.05\times10^{-34}\text{J}\cdot\text{s})^2}{2(.2\text{Kg})(1.5\text{m})^2}(1) \Rightarrow \boxed{E_1 = 1.2\times10^{-67}\text{J}}$$

$$\text{UNIT: } \frac{\text{J}^2\cdot\text{s}^2}{\text{Kg}\cdot\text{m}^2} = \frac{\text{J}(\text{Kg}\,\text{m}^2/\text{s}^2)\,\text{s}^2}{\text{Kg}\cdot\text{m}^2} = \text{J}$$

b) $K = E_1 = \frac{1}{2}MV^2 \Rightarrow V = \sqrt{\frac{2E_1}{M}} = \left(\frac{2\cdot1.2\times10^{-67}\text{J}}{.2\text{kg}}\right)^{1/2} \Rightarrow \boxed{V = 1.1\times10^{-33}\text{m/}}$

$t = ?$ CONSTANT $V \Rightarrow X = Vt$

$$\Rightarrow t = \frac{X}{V} = \frac{1.5\text{m}}{1.1\times10^{-33}\text{m/s}} \Rightarrow \boxed{t = 1.4\times10^{33}\text{S} = 4.3\times10^{25}\text{ YEARS!}}$$

c $E_2 - E_1 = ?$

$$E_2 - E_1 = \frac{\pi^2\hbar^2}{2ML^2}2^2 - \frac{\pi^2\hbar^2}{2ML^2}1^2 = \frac{\pi^2\hbar^2}{2ML^2}(4-1) = \frac{\pi^2\hbar^2}{2ML^2}(3)$$

$$= 1.2\times10^{-67}\text{J}(3) \Rightarrow \boxed{E_2 - E_1 = 3.6\times10^{-67}\text{J}}$$

d ΔE IS TOO SMALL TO BE MEASURED, QM IS NOT IMPORTANT.

40.11 ELECTRON IN BOX $L = 3\times10^{-10}$ m.

FIND λ AND p FOR $n=1,2,3$.

$$p=\hbar K. \quad K=\frac{n\pi}{L} \quad \Rightarrow p=\frac{\hbar n\pi}{L}$$

$$\lambda=\frac{h}{p}=\frac{h}{\hbar K}=\frac{h}{\frac{h}{2\pi}K}=\frac{2\pi}{K}$$ (JUST LIKE LIGHT!)

$$\Rightarrow \lambda=\frac{2\pi}{n\pi/L}=\frac{2L}{n}$$ → WAVELENGTH IS ALWAYS ON SAME ORDER AS BOX LENGTH.

a) $n=1$ $\Rightarrow \boxed{\lambda=2L=6\times10^{-10}\text{ m}}$

$$p=\frac{(1.05\times10^{-34}\text{ J.s})\,\pi}{3\times10^{-10}\text{ m}} \Rightarrow \boxed{p=1.1\times10^{-24}\text{ Kgm/s}}$$

b) $n=2$ $\boxed{\lambda=L=3\times10^{-10}\text{ m}}$

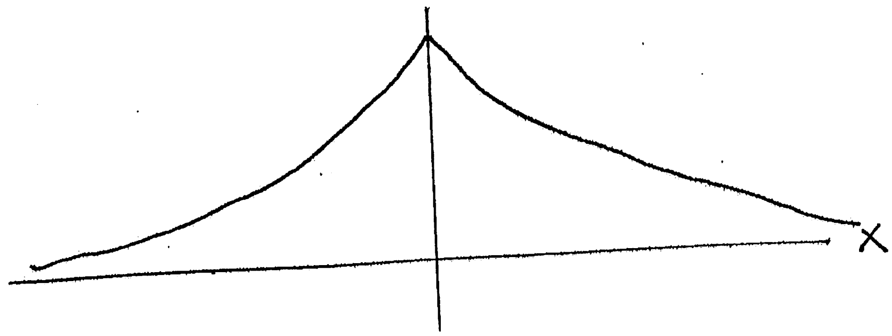
$$p=\frac{(1.05\times10^{-34}\text{ J.s})\,\pi\,(2)}{3\times10^{-10}\text{ m}} \Rightarrow \boxed{p=2.2\times10^{-24}\text{ Kgm/s}}$$

c) $n=3$ $\boxed{\lambda=\frac{2L}{3}=2\times10^{-10}\text{ m}}$

$$\boxed{p=3.3\times10^{-24}\text{ Kgm/s}}$$

40.43 $\quad \Phi = \begin{cases} e^{K'x} & x<0 \\ e^{-K'x} & x \geq 0 \end{cases}$ FOR $U=0$

a



b, c SHOW Φ SATISFIES SCHRÖDINGER EQN.

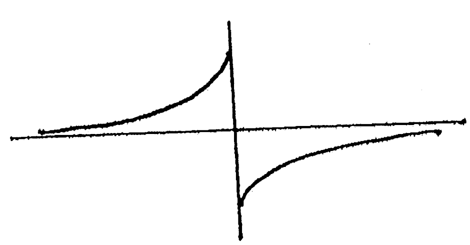
$U=0 \Rightarrow -\frac{\hbar^2}{2M}\frac{d^2\Phi}{dx^2} = E\Phi$

$\frac{d\Phi}{dx} = \pm K' e^{\pm K'x}$ $\left(\begin{array}{l} + \text{ FOR } x<0 \\ - \text{ FOR } x>0 \end{array}\right)$. $\frac{d^2\Phi}{dx^2} = (\pm K')^2 e^{\pm K'x}$

$(\pm K')^2 = (\pm 1)^2 K'^2 = K'^2 \Rightarrow \frac{d^2\Phi}{dx^2} = K'^2\Phi$

$\Rightarrow -\frac{\hbar^2 K'^2}{2M}\Phi = E\Phi \Rightarrow E = -\frac{\hbar^2 K'^2}{2M} \rightarrow$ HAS $E<0$

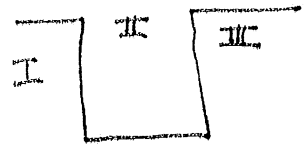
d THIS IS STILL NOT AN ACCEPTABLE SOLUTION BECAUSE $\frac{d\Phi}{dx}$ IS NOT CONTINUOUS AT $X=0$.



$\frac{d\Phi}{dx} = \begin{cases} K'e^{K'x} & x<0 \\ -K'e^{-K'x} & x\geq 0 \end{cases}$

40.44 Penetration depth η defined by $\psi(L+\eta) = \frac{1}{e}\psi(L)$

$$\eta = \frac{\hbar}{\sqrt{2m(U_0-E)}}$$



By the way this is very easy to show.

$$\psi_{III}(x) = Ge^{-\kappa x} \Rightarrow \psi(L+\eta) = \frac{1}{e}\psi(L) \Rightarrow Ge^{-\kappa'(L+\eta)} = e^{-1}Ge^{-\kappa' L}$$

$$\Rightarrow e^{-\kappa'(L+\eta)} = e^{-1}e^{-\kappa' L} \Rightarrow e^{-\kappa' L}e^{-\kappa'\eta} = e^{-1}e^{-\kappa' L}$$

$$\Rightarrow e^{-\kappa'\eta} = e^{-1} \Rightarrow \kappa'\eta = 1 \Rightarrow \eta = \frac{1}{\kappa'}. \quad \kappa' = \sqrt{\frac{2m(U_0-E)}{\hbar^2}}$$ gives EQN Above.

a) electron WITH $E = 13eV$, in $U_0 = 20eV$

Let's just find κ', it's easier actually. Note units: $\frac{kg \cdot eV}{(eV \cdot s)(J s)} = \frac{kg}{J \cdot s^2} = \frac{kg}{kg \cdot m^2/s^2 \cdot s^2} = \frac{1}{m^2}$ If we use $\hbar^2 = \hbar$ in $eV \times \hbar$ in J.

$$\kappa' = \sqrt{\frac{2(9.11\times10^{-31}kg)(7eV)}{(6.6\times10^{-16}eV.s)(1.05\times10^{-34}J.s)}} = 1.357\times10^{10}/m$$

$$\Rightarrow \boxed{\eta = \frac{1}{1.357\times10^{10}/m} = 7.37\times10^{-11}m = 73.7pm}$$

b) Proton WITH $E = 20MeV$ in $U_0 = 30MeV$

$$\Rightarrow U_0 - E = 10MeV = 1\times10^{7}eV$$

$$\Rightarrow \kappa' = \left[\frac{2(1.67\times10^{-27}kg)(1\times10^{7}eV)}{(6.6\times10^{-16}eV.s)(1.05\times10^{-34}Js)}\right]^{1/2} = 6.94\times10^{14}/m$$

$$\Rightarrow \boxed{\eta = \frac{1}{6.94\times10^{14}/m} = 1.44\times10^{-15}m = 1.44\,fm}$$

40.46 ELECTRON WITH $E = 5.5eV$ HITS $U_0 = 10eV$ BARRIER. WHAT IS WIDTH IF $T = .1\% = .001$

$$T \ll 1 \Rightarrow T = 16\left(\frac{E}{U_0}\right)\left(1-\frac{E}{U_0}\right)e^{-2k'L} \quad k' = \left[\frac{2M(U_0-E)}{\hbar^2}\right]^{1/2}$$

$$k' = \left[\frac{2(9.11\times10^{-31}kg)(10eV-5.5eV)\cdot1.6\times10^{-19}J/eV}{(1.05\times10^{-34}J\cdot s)^2}\right]^{1/2} = 1.09\times10^{10}/m$$

$$\Rightarrow .001 = 16\left(\frac{5.5}{10}\right)\left(1-\frac{5.5}{10}\right)e^{-2(1.09\times10^{10}/m)L}$$

$$\Rightarrow .001 = 3.96\,e^{-2(1.09\times10^{10}/m)L} \Rightarrow -2(1.09\times10^{10}/m)L = \ln\left(\frac{.001}{3.96}\right)$$

$$\Rightarrow \boxed{L = 3.8\times10^{-10}m = .38nm}$$

40.48 $M = .02kg$ MASS ON SPRING WITH $f = 1.5Hz$, $V_{MAX} = .36m/s$

LOOKING BACK TO CH. 13, WE FIND $\omega = 2\pi f$, $\omega = \sqrt{\frac{K}{M}}$, $E = \frac{1}{2}KA^2$, $V_{MAX} = \omega A$

$$\Rightarrow \omega = 2\pi(1.5Hz) = 3\pi\ RAD/s, \quad A = \frac{V_{MAX}}{\omega} = \frac{.36m/s}{3\pi/s} = \frac{.12}{\pi}m$$

$$E = \frac{1}{2}KA^2 = \frac{1}{2}M\omega^2A^2 = \frac{1}{2}(.02kg)(3\pi/s)^2\left(\frac{.12}{\pi}m\right)^2 \Rightarrow \boxed{E = 1.296\times10^{-3}J}$$

$$E = (n+\tfrac{1}{2})\hbar\omega \Rightarrow 1.296\times10^{-3}J = (n+\tfrac{1}{2})(1.05\times10^{-34}J\cdot s)(3\pi/s)$$

$$\Rightarrow n = 1.3\times10^{30} - \tfrac{1}{2} \Rightarrow \boxed{n = 1.3\times10^{30}}$$

b) $\Delta E = E_{n+1} - E_n = \hbar\omega(n+1+\tfrac{1}{2}) - \hbar\omega(n+\tfrac{1}{2}) = \hbar\omega$

$$\Delta E = (1.05\times10^{-34}J\cdot s)(3\pi/s) \Rightarrow \boxed{\Delta E = 9.9\times10^{-34}J = 6\times10^{-15}eV}$$

NOT DETECTABLE!