PHYS 262, HW#5: 37.29, 37.44, 37.47, 37.66, 37.69

37.29 a) AT WHAT SPEED IS REL. MOMENTUM TWICE THE CLASSICAL VALUE?

$$P = \gamma M_0 V = 2 M_0 V$$

$$\Rightarrow \gamma = 2 \Rightarrow \frac{1}{\sqrt{1-V^2/C^2}} = 2 \Rightarrow 1 - V^2/C^2 = \left(\tfrac{1}{2}\right)^2$$

$$\Rightarrow V^2/C^2 = 1 - \left(\tfrac{1}{2}\right)^2 = 3/4 \Rightarrow \boxed{V = \sqrt{3/4}\, C = .866C}$$

b) AT WHAT SPEED IS FORCE NEEDED TO ACCELERATE TWICE THAT NEEDED TO ACCELERATE FROM REST.

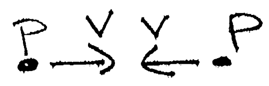
$$F = \gamma^3 M_0 a . \quad \text{AT REST, } \gamma = 1 \Rightarrow \gamma^3 M_0 a = 2 (M_0 a)$$

$$\Rightarrow \gamma^3 = 2 \Rightarrow \gamma = 2^{1/3} \Rightarrow \frac{1}{\sqrt{1-V^2/C^2}} = 2^{1/3} \Rightarrow 1 - V^2/C^2 = \left(\tfrac{1}{2^{1/3}}\right)^2$$

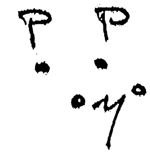
$$\Rightarrow 1 - V^2/C^2 = 2^{-2/3} \Rightarrow V^2/C^2 = 1 - 2^{-2/3}$$

$$\Rightarrow \boxed{V = \sqrt{1-2^{-2/3}}\, C = .608c}$$

37.44



BEFORE



AFTER

REST MASS:

PROTON: $M_p = 1.67 \times 10^{-27}$ kg

η^0: $M_\eta = 9.75 \times 10^{-28}$ kg

a) FIND V OF PROTONS:

CONSERVATION OF ENERGY TELLS US THAT TOTAL ENERGY, E, BEFORE AND AFTER REMAINS UNCHANGED.

MOVING $\Rightarrow E = \gamma M_o C^2$, AT REST $\Rightarrow E = M_o C^2$

BEFORE: $E_{TOTAL} = E_{P_1} + E_{P_2} = \gamma_1 M_p C^2 + \gamma_2 M_p C^2$.

$\gamma_1 = \dfrac{1}{\sqrt{1 - V_1^2/C^2}}$, $\gamma_2 = \dfrac{1}{\sqrt{1 - V_2^2/C^2}}$. V_1 = Proton #1 speed, V_2 = Proton #2 speed.

$V_1 = V_2 = V \Rightarrow \gamma_1 = \gamma_2 = \gamma = \dfrac{1}{\sqrt{1 - V^2/C^2}} \Rightarrow E_{TOTAL} = 2\gamma M_p C^2$

AFTER: $E_{TOTAL} = M_p C^2 + M_p C^2 + M_\eta C^2 = 2 M_p C^2 + M_\eta C^2$

CONSERVATION $\Rightarrow 2\gamma M_p C^2 = 2 M_p C^2 + M_\eta C^2$

$$\Rightarrow \gamma = 1 + \frac{M_\eta}{2M_p} = 1 + \frac{9.75 \times 10^{-28}}{2(1.67 \times 10^{-27})} = 1.292$$

$$\Rightarrow \frac{1}{\sqrt{1 - V^2/C^2}} = 1.292 \Rightarrow V = \sqrt{1 - \left(\tfrac{1}{1.292}\right)^2}\, C = \sqrt{.4}\, C$$

$$\Rightarrow \boxed{V = .632\, C}$$

b) FIND K OF Protons.

$$K = (\gamma - 1) M_p c^2 = (1.292 - 1)(1.67\times10^{-27}\,kg)(3\times10^8\,m/s)^2$$

$$\Rightarrow \boxed{K = 4.4\times10^{-11}\,J}$$

Express ANSWER IN TERMS OF MeV.

→ THIS IS KIND OF JUMPING AHEAD, BUT eV stands for electron Volts.

$1\,eV = 1.6\times10^{-19}\,J.$ $1\,MeV = 1\times10^6\,eV$ (ONE Million eV).

$$4.4\times10^{-11}\,J \times \frac{1\,eV}{1.6\times10^{-19}\,J} = 2.74\times10^8\,eV = 274\times10^6\,eV \Rightarrow \boxed{K = 274\,MeV}$$

c) WHAT IS REST ENERGY OF η^0 IN MeV?

$$M_\eta c^2 = (9.75\times10^{-28}\,kg)(3\times10^8\,m/s^2)^2 = 8.78\times10^{-11}\,J$$

$$8.78\times10^{-11}\,J \times \frac{1\,eV}{1.6\times10^{-19}\,J} = 5.48\times10^8\,eV = 548\times10^6\,eV$$

$$\Rightarrow \boxed{M_\eta c^2 = 548\,MeV}$$

d) NOTICE THAT $M_\eta c^2 = 2K$! THE KINETIC ENERGY OF THE PROTONS IS CONVERTED INTO THE η^0's MASS. MASS IS ENERGY.

37.47 SUN PRODUCING ENERGY AT A RATE OF 3.8×10^{26} WATT.

a) HOW MANY Kilograms of MATTER does SUN lose every SECOND?

$$P = \frac{dE}{dt} \Rightarrow dE = P\,dt \Rightarrow \Delta E = P\Delta t = (3.8\times10^{26}\,\text{watt})(1\,\text{s})$$
$$= 3.8\times10^{26}\,\text{J}$$

$$\Delta E = \Delta M c^2 \Rightarrow 3.8\times10^{26}\,\text{J} = (\Delta M)(3\times10^8\,\text{m/s})^2$$
$$\Rightarrow \boxed{\Delta M = 4.22\times10^{9}\,\text{kg}}$$

b) HOW MANY Tons?

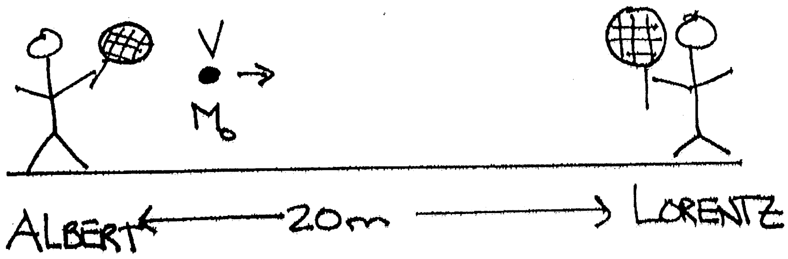
$$4.22\times10^9\,\text{Kg} \times \frac{2.205\,\text{lb}}{\text{Kg}} \times \frac{\text{ton}}{2000\,\text{lb}} = \boxed{4.655\times10^6\,\text{ton}}$$
$$= 4.655\,\text{M ton}$$

c) HOW LONG FOR SUN TO USE UP ITS MASS?

$$M_{SUN} = 1.99\times10^{30}\,\text{kg} \Rightarrow \frac{1.99\times10^{30}\,\text{kg}}{4.22\times10^9\,\text{kg/s}} = 4.716\times10^{20}\,\text{s}$$

$$4.716\times10^{20}\,\text{s} \times \frac{\text{h}}{3600\,\text{s}} \times \frac{\text{day}}{24\,\text{h}} \times \frac{\text{year}}{365\,\text{day}} = \boxed{1.5\times10^{13}\,\text{years}}$$

37.66



$M_0 = .058\ Kg$

a) IF $V = 80\,m/s$, WHAT IS KINETIC ENERGY?

AT SUCH A SMALL SPEED, WE CAN USE $K = \frac{1}{2}MV^2 = \frac{1}{2}(.058\,Kg)(80\,m/s)^2$

$$\Rightarrow \boxed{K = 185.6\,J}$$

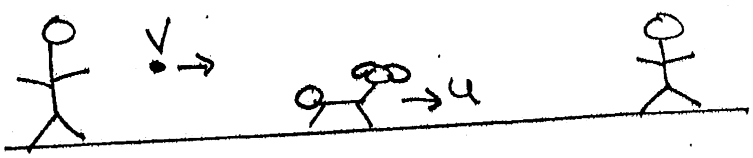
b) IF $V = 1.8\times10^8\,m/s$, WHAT IS KINETIC ENERGY?

HERE, WE NEED $K = (\gamma-1)M_0C^2$. $\gamma = \dfrac{1}{\sqrt{1-\left(\frac{1.8}{3}\right)^2}} = 1.25$

$\hookrightarrow \dfrac{1.8\times10^8}{3\times10^8} = \dfrac{1.8}{3}$

$$\Rightarrow K = .25(.058\,Kg)(3\times10^8\,m/s)^2 \Rightarrow \boxed{K = 1.3\times10^{15}\,J}$$

c) RABBIT RUNS FROM Albert TO LORENTZ AS BAll goes FROM Albert TO LORENTZ.



LET S BE Albert/LORENTZ FRAME.
S' BE RABBIT'S FRAME
$\Rightarrow u = 2.2\times10^8\,m/s \rightarrow$ RABBIT'S SPEED

Albert AND LORENTZ SEE $V = 1.8\times10^8\,m/s$ → SAME DIRECTION AS RABBIT.

CAll RABBIT'S DIRECTION X $\Rightarrow V_x = 1.8\times10^8\,m/s$, $u = 2.2\times10^8\,m/s$, $V_x' = ?$

$$V_x' = \frac{V_x - u}{1 - \frac{uV_x}{C^2}} = \frac{1.8\times10^8\,m/s - 2.2\times10^8\,m/s}{1 - \frac{(2.2)(1.8)}{3^2}} \Rightarrow \boxed{V_x' = -7.14\times10^7\,m/s}$$

d WHAT DISTANCE DOES RABBIT MEASURE FROM ALBERT TO LORENTZ?

A METER STICK FROM ALBERT TO LORENTZ DOES NOT MOVE IN THEIR FRAME OF REFERENCE (S) $\Rightarrow$ THEY MEASURE PROPER LENGTH. $\rightarrow$ $L_o = 20m$

RABBIT'S MEASURED LENGTH: $L = \frac{L_o}{\gamma_R}$ ($\gamma_R = \gamma$ FOR RABBIT'S SPEED)

$$\gamma_R = \frac{1}{\sqrt{1-\left(\frac{2.2}{3}\right)^2}} = 1.47 \Rightarrow \boxed{L = \frac{20m}{1.47} = 13.6m}$$

e) HOW LONG DOES IT TAKE RABBIT TO RUN ACCORDING TO PLAYERS? RABBIT MEASURES PROPER TIME.*

Players MEASURE A PROPER DISTANCE L_o

$$\Rightarrow \Delta t = \frac{L_o}{u} = \frac{20m}{2.2\times10^8 m/s} \Rightarrow \boxed{\Delta t = 9.09\times10^{-8} s}$$

f) $\Delta t = \gamma \Delta t_o \Rightarrow \Delta t_o = \frac{\Delta t}{\gamma_R} = \frac{9.09\times10^{-8}s}{1.47} \Rightarrow \boxed{\Delta t_o = 6.18\times10^{-8}s}$

OR WE COULD USE $\Delta t_o = \frac{L}{u} = \frac{13.6m}{2.2\times10^{+8} m/s} = 6.18\times10^{-8}s$

(* BECAUSE THE EVENT IS THE RABBIT going from ALBERT TO LORENTZ.)

37.69

V_{rocket}	t_{earth},	t_{rocket},	E (J),	E (%)
.5c	1000 yr	866 yr	1.04×10^{20}	1040%
.99c	505 yr	71.2 yr	6.38×10^{20}	6380%
.9999c	500 yr	7.07 yr	6.36×10^{21}	63641% → THAT'S A lot

EVENT = Rocket TRAVELS to Betelgeuse ⇒ $t_{Rocket} = \Delta t_0$.

⇒ $t_{earth} = \Delta t$ ⇒ $t_{earth} = \gamma\, t_{Rocket}$ ⇒ $t_{Rocket} = \frac{1}{\gamma} t_{earth}$.

Earth MEASURES l_0 ⇒ $t_{earth} = \frac{500 \text{ light-years}}{V}$

Moving Rocket ⇒ $E = \gamma M_0 c^2 = \gamma (1000\text{kg}) c^2 = \gamma (9\times10^{19}\text{J})$

a) $V = .5c$ ⇒ $\gamma = \frac{1}{\sqrt{1-.5^2}} = 1.1547$, $t_{earth} = \frac{500\text{ l-yr}}{.5(\text{light})} = 1000$ year.

⇒ $t_{rocket} = \frac{1000\text{yr}}{1.1547} = 866\text{ yr}$, $E = 1.04\times10^{20}\text{J}$

$\% = \frac{1.04\times10^{20}\times100}{1\times10^{19}} = 10.4\times100 = 1040\%$

b) $V = .99c$ ⇒ $t_{earth} = \frac{500}{.99} = 505$, $\gamma = \frac{1}{\sqrt{1-.99^2}} = 7.088, \ldots$ (see table)

c) $V = .9999c$, $\gamma = 70.712$