PHYS 262, HW#4

37.5, 37.8, 37.13, 37.51, 37.54, 37.71

37.8

YOU SEE SPACE SHIP'S SEARCH LIGHT FOR .19s. SOMEONE ON SPACESHIP SEES SEARCH LIGHT FOR 12ms = 1.2×10^{-2}s

a) WHICH TIME IS PROPER TIME?

THE EVENT WHICH OCCURS IS THAT THE SPACESHIP TURNS ON ITS LIGHT. IF WE PUT A WATCH ON THE LIGHT, YOU SEE WATCH MOVING SO YOU MEASURE Δt. SOMEONE ON SPACESHIP SEES STATIONARY WATCH SO THEY MEASURE Δt_0.

$\Rightarrow \Delta t = .19\text{s},\ \Delta t_0 = 1.2\times10^{-2}\text{s}$ (WE COULD ALSO USE THE FACT THAT THE PROPER TIME IS ALWAYS SMALLER THAN THE DILATED TIME.)

$$\Delta t = \gamma \Delta t_0 \Rightarrow .19\text{s} = \gamma\,(1.2\times10^{-2}\text{s}) \Rightarrow \gamma = 15.83$$

$$\gamma = \frac{1}{\sqrt{1-V^2/c^2}} \Rightarrow 1 - V^2/c^2 = \frac{1}{(15.83)^2} \Rightarrow V = .998c$$

$\underline{37.5}$ $\Delta t_0 = 2.6 \times 10^{-8}$ s (SINCE MEASURED IN REST FRAME)

$\Delta t = 4.2 \times 10^{-7}$ s.

a) FIND SPEED AS A FRACTION OF C.

$V = ?$

$$\Delta t = \gamma \Delta t_0 \Rightarrow \gamma = \frac{\Delta t}{\Delta t_0} = \frac{4.2 \times 10^{-7}\,s}{2.6 \times 10^{-8}\,s} = 16.154$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/C^2}} \Rightarrow \gamma^2 = \frac{1}{1 - V^2/C^2} \Rightarrow 1 - V^2/C^2 = 1/\gamma^2 \Rightarrow V^2/C^2 = 1 - \frac{1}{\gamma^2}$$

$$\Rightarrow \frac{V}{C} = \sqrt{1 - 1/\gamma^2} \Rightarrow V = \sqrt{1 - 1/\gamma^2}\, C = \sqrt{1 - \frac{1}{16.154^2}}\, C$$

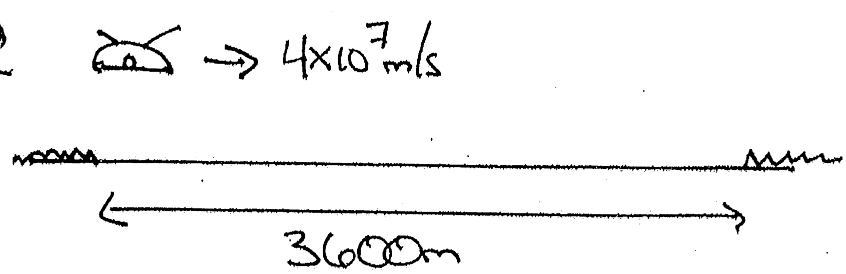
$$\Rightarrow \boxed{V = .998c}$$

b) WHAT DISTANCE TRAVELLED IN LAB FRAME.

IN LAB, WE SEE π^- WITH $V = .998c$ MOVING FOR 4.2×10^{-7} s

$$\Rightarrow d = (.998)(3 \times 10^8\, m/s)(4.2 \times 10^{-7}\, s) \Rightarrow \boxed{d = 125.76m = 126m}$$

37.13



a) WHAT IS RUNWAY'S LENGTH AS MEASURED BY SPACESHIP'S PILOT?

EARTH MEASURES PROPER LENGTH. BECAUSE A 3600m LONG STICK FROM ONE END OF RUNWAY TO OTHER IS STATIONARY ON EARTH.

$\Rightarrow L_0 = 3600\text{m}. \quad L = ? \quad L = \frac{L_0}{\gamma}$

$\gamma = \dfrac{1}{\sqrt{1-\left(\frac{4\times10^7}{3\times10^8}\right)^2}} = \dfrac{1}{\sqrt{1-\left(\frac{.4}{3}\right)^2}} = 1.009$

$\Rightarrow L = \dfrac{3600\text{m}}{1.009} \Rightarrow \boxed{L = 3567.9\text{m} = 3570\text{m}}$

b) HOW LONG DOES EARTHLING MEASURE FOR SPACESHIP TO FLY?

ON EARTH SPACESHIP TRAVELS 3600m WITH CONSTANT 4×10^7 m/s SPEED

$\Rightarrow t_{earth} = \dfrac{3600\text{m}}{4\times10^7\text{m/s}} \Rightarrow \boxed{t_{earth} = 9\times10^{-5}\text{s}}$

c) HOW LONG DOES PILOT MEASURE?

PILOT SEES 3611m LONG RUNWAY TRAVELLING AT 4×10^7m

$\Rightarrow t_{pilot} = \dfrac{3567.9\text{m}}{4\times10^7\text{m}} \Rightarrow \boxed{t_{pilot} = 8.92\times10^{-5}\text{s}}$

Notice $\dfrac{t_{earth}}{t_{pilot}} = \gamma$

$t_{earth} = \gamma\, t_{pilot}$

$\Delta t = \gamma\, \Delta t_0$

37.51

THE DRAWING IS A LITTLE CONFUSING! I THINK THEY MEAN:

FEDERATION EMPIRE

$a = 1.4b$

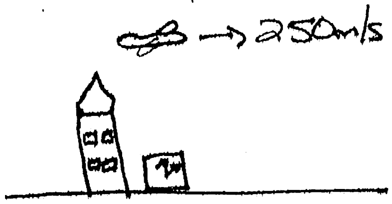
SO WE WANT TO KNOW AT WHAT SPEED WILL THE FEDERATION'S SHIP BE CONTRACTED BY $\frac{1}{1.4} = .714$

$$L = \frac{L_0}{\gamma} \Rightarrow .714 L_0 = \frac{L_0}{\gamma} \Rightarrow \gamma = 1.4 \Rightarrow \frac{1}{\sqrt{1 - v^2/c^2}} = 1.4$$

$$\Rightarrow \sqrt{1 - v^2/c^2} = \frac{1}{1.4} \Rightarrow 1 - v^2/c^2 = \left(\frac{1}{1.4}\right)^2 = .510 \Rightarrow v = \sqrt{1 - .510}\, c$$

$$= .7c$$

37.54



Clock on Ground gives elapsed time of 4.00 hour.

By how much will plane and ground clock differ?

Since the elapsed time we are measuring is the time for the airplane to travel to and fro ~~Fr~~ New York, people on the airplane measure the proper, i.e., shorter time.

$\Rightarrow \Delta t - \Delta t_0 = ?$, $\Delta t = 4.00$ hour. $V = 250 m/s$.

$$\Delta t = (\Delta t_0)\gamma = \Delta t_0 \frac{1}{\sqrt{1 - V^2/c^2}} \Rightarrow \Delta t_0 = \sqrt{1 - V^2/c^2}\,\Delta t.$$

Since $V \ll c$, we can use binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

If $x \ll 1$ then $(1+x)^n \approx 1 + nx$.

$$\sqrt{1 - V^2/c^2} = (1 - V^2/c^2)^{1/2} \approx 1 - \tfrac{1}{2}V^2/c^2 \Rightarrow \Delta t_0 = (1 - \tfrac{1}{2}V^2/c^2)\Delta t$$

$$\Rightarrow \Delta t_0 = \Delta t - \tfrac{1}{2}V^2/c^2\,\Delta t \Rightarrow \Delta t - \Delta t_0 = \tfrac{1}{2}V^2/c^2\,\Delta t.$$

$$\Delta t = 4h = 14400s \Rightarrow \Delta t - \Delta t_0 = \frac{(250 m/s)^2}{2(3\times10^8 m/s)^2}(14400s) = 5\times10^{-9} s$$

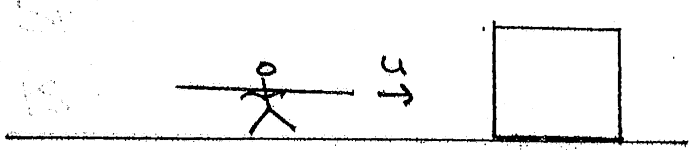
$$= 5 ns$$

37.71 THE SOLUTION TO THIS PARADOX (LIKE SO MANY) IS SIMULTANEITY. WHAT OCCURS AT THE SAME TIME IN ONE FRAME IS NOT SIMULTANEOUS IN ANOTHER. SO THE POLE CAN FIT IN ONE FRAME AND NOT IN ANOTHER WITH NO PARADOX.

TO FLESH THIS OUT, LET'S MAKE S = FRAME WHERE BARN IS STATIONARY AND S' = FRAME WHERE RUNNER IS STATIONARY.

TO SIMPLIFY A BIT, LET'S ALSO ASSUME RUNNER HAS A SPEED U SUCH THAT IN S, THE POLE JUST FITS INSIDE THE BARN

$\Rightarrow L_0 = 6m,\ L = 5m.\ L = \frac{L_0}{\gamma} \Rightarrow \gamma = \frac{6}{5} = 1.2 \Rightarrow u = .553c.$



THE TWO SIMULTANEOUS EVENTS IN THE S FRAME ARE

LEFT END OF POLE IS AT LEFT END OF BARN $\Rightarrow x_1 = 0,\ t_1 = 0$

RIGHT END OF POLE IS AT RIGHT END OF BARN $\Rightarrow x_2 = 5m,\ t_2 = 0$

LORENTZ TRANSFORM: $t' = \gamma\left(t - \frac{ux}{c^2}\right) \Rightarrow$

$t_1' = \frac{6}{5}\left(0 - \frac{.553c(0)}{c^2}\right) = 0 \quad t_2 = \frac{6}{5}\left(0 - \frac{.553c(5m)}{c^2}\right) = -1.1\times10^{-8}s = -11\times10^{-9}s$

$\therefore$ RUNNER SAYS, THE RIGHT END OF MY POLE HIT THE RIGHT END OF THE BARN 11ns BEFORE THE LEFT END HIT THE LEFT END. OF COURSE I'M NOT INSIDE THE BARN!