Physics 262
Extra Credit
Spring 2011

One of the ways to see why we need Einstein's theory of special relativity is to find the "flaw" in Maxwell's equations: they do not always correctly predict the interrelationship between electric and magnetic fields. We know that induction states that when an electric field changes with time that there is a magnetic field created. Ampere's law (for the $\vec{\mathbf{J}} = 0$ situation) gives the equation

$$\vec{\nabla} \times \vec{\mathbf{B}} = \epsilon_0 \mu_0 \left(\frac{\partial \vec{\mathbf{E}}}{\partial t} \right).$$

But does this give a prediction that agrees with what we observe experimentally?

To check, consider the simple situation of a point charge q moving with a velocity $\vec{\mathbf{v}}$. We learned in Physics II that experiments give the magnetic field created by this point charge as

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \left(\frac{q \vec{\mathbf{v}} \times \vec{\mathbf{r}}}{r^3} \right) \tag{1}$$

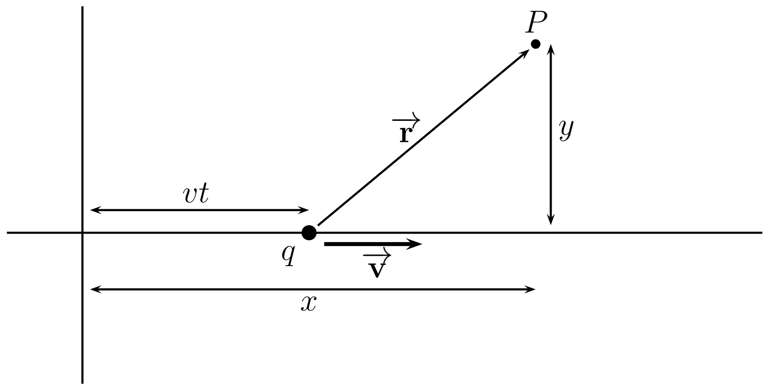
where $\vec{\mathbf{r}}$ is the position vector from the point charge to a point P.

We also learned Coulomb's law in Physics II. It gives the equation for the electric field created by a point charge,

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \left(\frac{q \vec{\mathbf{r}}}{r^3} \right)$$

where $\vec{\mathbf{r}}$ is again the position vector (and I have put it in a vector form more conducive to this exercise).

Consider a point particle q traveling in the positive x-direction with a speed v. The distance traveled by this particle in a time t would therefore be vt. For simplicity, assume the point P of interest is in the same plane as particle and so $\vec{\mathbf{r}} = (x - vt)\,\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$.



Tasks for Extra-Credit

1. Using equation 1, find the Cartesian co-ordinate equation for the magnetic field at point P created by this particle.

2. Take the curl of this magnetic field (again in Cartesian co-ordinates).

3. Using Coulomb's law, find the equation for the electric field at point P.

4. Take the partial time derivative of this electric field.

5. Does Ampere's law hold true?