Physics 262
Exam 5

Please answer any four of the following five questions. Each question is worth five points. Partial credit will be awarded for any **attempted** problem.

1. According to Bohr model, what frequency of light would be emitted by an electron moving from the $n = 5$ to the $n = 2$ level of the singly ionized helium atom? (That is, the helium atom with a single electron).

Bohr model, $E_n = -\frac{Z^2 (13.6\,eV)}{n^2}$

Helium $\Rightarrow Z=2 \Rightarrow E_n = -\frac{54.4\,eV}{n^2}$

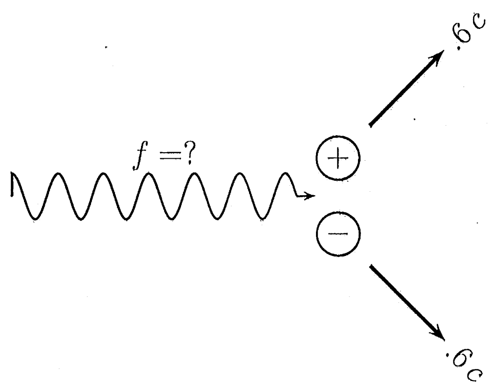
$\Rightarrow E_5 = -2.176\,eV, \quad E_2 = -13.6\,eV$

$\Delta E = -13.6\,eV - (-2.176\,eV) = -11.424\,eV$

$\Rightarrow E_{photon} = +11.424\,eV = hf$

$\Rightarrow f = \frac{11.424\,eV}{4.136\times10^{-15}\,eV\cdot s} = \underline{2.762\times10^{15}\,Hz} \leftarrow$ UV/purple

2. As introduced in the previous exam, the positron is a particle that is identical in every way to an electron except that it has positive charge. Positrons are created when high energy photons convert themselves into a positron and electron pair. (One of each in order to conserve charge). What frequency must the photon have in order to create a positron and electron, each with speed .6c? Use 0.511 MeV/c^2 for the rest mass of the electron and positron.



to create two identical mass particles with $v = .6c$

$$\Rightarrow E = 2\gamma m_0 c^2 \qquad \gamma = \frac{1}{\sqrt{1-.6^2}} = 1.25$$

$$\Rightarrow E = 2(1.25)(.511\,MeV/\cancel{c^2})\cancel{c^2} = 1.2775\,MeV$$

$$= 1.2775 \times 10^6\,eV$$

$$E = hf \Rightarrow f = \frac{1.2775 \times 10^6\,eV}{4.136 \times 10^{-15}\,eV \cdot s} = 3.09 \times 10^{20}\,Hz$$

Very high frequency
GAMMA RAY

3. One practical application of laser technology is the removal of tatoos. The "removal" is actually the breaking apart of the pigment molecules inside the tatoo. The broken molecules are then slowly absorbed by the body. For complete removal, multiple treatments are required. A quick search of the available literature reveals that red pigments are best removed using a 532 nm wavelength laser, which itself is green in color. (Other color pigments require different wavelengths, but we're not concerned with them today).

   In order to break apart the pigment molecules, a laser pulse lasting around 25 ns is directed onto the skin. In one study, the laser beam had a power of 300 $Watt$ and created a 3-mm radius circular beam. Assuming there are 5×10^{10} pigment molecules per square meter of skin and that a single laser pulse can break down 1/4 of the total, find the energy (in Joules) needed to break apart a single tatoo molecule.

Single pulse destroys $\frac{1}{4}(5\times10^{10})\,\frac{\text{molecules}}{m^2} = 1.25\times10^{10}\,\frac{\text{molecules}}{m^2}$

So Find $N = \#\text{photons}/m^2$ $\Rightarrow$ $\frac{N}{1.25\times10^{10}} = \frac{\#\text{photons}}{\text{molecule}}$

$E = hf = \text{energy/photon} \Rightarrow \left(\frac{N}{1.25\times10^{10}}\right)hf = \text{energy/molecule}$

$$\Rightarrow \left(\frac{N}{1.25\times10^{10}}\right)\frac{hc}{\lambda} = E_{\text{molec.}} = ?$$

$$\text{Watt} = J/s \Rightarrow \frac{(300\,\text{watt})(25\times10^{-9}s)}{\pi(3\times10^{-3}m)^2} = .2653\,J/m^2 \Rightarrow N = \frac{.2653\,J/m^2}{(hc/\lambda)}$$

Area of circle

$$\Rightarrow E_{\text{molc}} = \frac{\left(\frac{.2653\,J}{(hc/\lambda)}\right)\frac{hc}{\lambda}}{1.25\times10^{10}} = \underline{2.122\times10^{-11}\,J/\text{molecules}}$$

For Completeness: $N = 7\times10^{17}\,\frac{\text{photons}}{m^2}$ $\quad \frac{N}{1.25\times10^{10}} = 5.67\times10^{7}$ photons

those molecules are tough!

4. A photoelectric experiment is performed on a metal whose minimum frequency is f_0. When using light of frequency $3f_0$ the stopping potential is measured to be $5.4\,V$. What is the work function for this metal in electron volts?

$$eV_0 = hf - \phi$$

when $f = f_0$, $V_0 = 0 \Rightarrow hf_0 = \phi$

$f = 3f_0$:

$$eV_0 = h(3f_0) - \phi \Rightarrow eV_0 = 3hf_0 - \phi$$

$V_0 = 5.4V \Rightarrow eV_0 = 5.4eV$

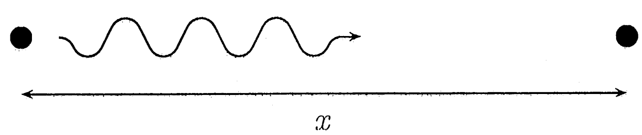
$\Rightarrow 5.4eV = 3hf_0 - \phi$ $\quad hf_0 = \phi$

$\Rightarrow 5.4eV = 3\phi - \phi = 2\phi \Rightarrow \boxed{\phi = 2.7eV}$

IN CASE, you care: $f_0 = \dfrac{\phi}{h} = \dfrac{2.7eV}{4.136\times10^{-15}\,eV\cdot s}$

$$f_0 = 6.528\times10^{14}\,Hz$$

5. According to the quantum theory of electricity and magnetism (called quantum field theory), the electric force is created when charged particles exchange "virtual" photons. These are undetectable photons which exist within the time limitations set by the uncertainty principle.



Two electrons are separated by a distance x as shown above. How much energy (as a function of x) can a virtual photon traveling between two electrons have?

The relationship between force and energy is fairly simple in one dimension: $F = -\frac{dE}{dx}$. Find the force F_V exerted by a virtual photon (again as a function of x).

Coulomb's law is

$$F_C = \frac{k|q_1||q_2|}{r^2}$$

where $k = 8.99 \times 10^9\, N \cdot m^2/C^2$. Show that for two electrons a distance x apart from each other that

$$F_C = \left(\frac{1}{137}\right) F_V.$$

This fraction, (1/137), is called the relative strength of the electromagnetic force (it's about 137 times weaker than the nuclear strong force - which we haven't discussed yet).

Energy: Uncertainty, $\Delta E \cdot \Delta t \geq \hbar$, $\Rightarrow$ min. $\Delta E \cdot \Delta t = \hbar$

photon traveling at speed of Light $\Rightarrow \Delta t = \frac{x}{c} \Rightarrow \Delta E\left(\frac{x}{c}\right) = \hbar$

$\Rightarrow \Delta E = \frac{\hbar c}{x}$. take ΔE as a measure of E $\Rightarrow \boxed{E = \frac{\hbar c}{x}}$

$$F_V = -\frac{dE}{dx} = -\hbar c \frac{d}{dx}\left(x^{-1}\right) = \frac{-\hbar c(-1)}{x^2} = \boxed{+\frac{\hbar c}{x^2} = F_V}$$

inverse square law!

$F_C = \frac{ke^2}{x^2}$ for 2 electrons, x apart

$$V = \frac{\hbar c}{x^2} \qquad F_c = \frac{Ke^2}{x^2}$$

$$\hookrightarrow \frac{1}{x^2} = \frac{F_V}{\hbar c}$$

$$\Rightarrow F_c = Ke^2\left(\frac{F_V}{\hbar c}\right) = \left(\frac{Ke^2}{\hbar c}\right) F_V$$

$$\frac{Ke^2}{\hbar c} = \frac{(8.99\times10^9)(1.6\times10^{-19}C)^2}{(1.057\times10^{-34})(3\times10^8)} = .00726 \approx \underline{.0073}$$

$$\frac{1}{137} = .0073 \Rightarrow F_c = \left(\frac{1}{137}\right) F_V$$

its : $$\frac{N\cdot m^{\cancel{2}}/\cancel{C^2}\cdot\cancel{C^2}}{(J\cdot\cancel{s})(\cancel{m}/\cancel{s})} = \frac{N\cdot m}{J} = \frac{J}{J} = \underline{1}$$