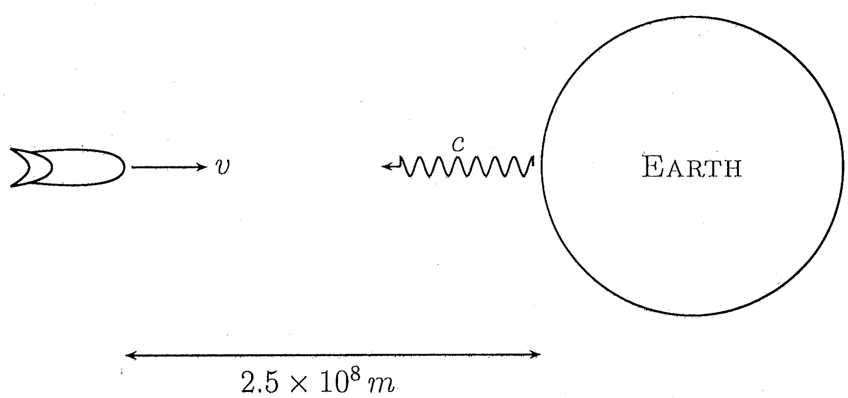
PHYSICS 262
EXAM 4

Please answer any four of the following five questions. Each question is worth five points. Partial credit will be awarded for any **attempted** problem.

1. A rocket is traveling towards earth with a speed of $v = 0.6\,c$ (relative to Earth). When the rocket is $2.5 \times 10^8\,m$ away (as measured from Earth), a radio transmitter on Earth sends out a welcome message. How long does it take the radio signal to reach the rocket as measured in earth's frame, as well as in the rocket's frame?



EVENT: LIGHT TRAVELS TO SPACESHIP $\Rightarrow$ SPACESHIP MEASURES PROPER TIME.

TO FIND DILATED TIME. ON EARTH: SPACESHIP DISTANCE + light = $2.5\times10^8 m$

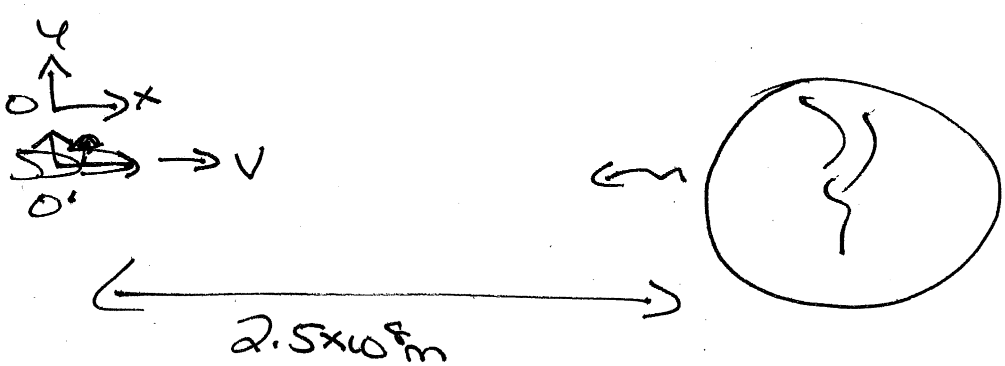
$\Rightarrow V\Delta t + C\Delta t = 2.5\times10^8 m \Rightarrow (V+C)\Delta t = 2.5\times10^8 m$

$\Rightarrow (1.6c)\Delta t = 2.5\times10^8 m \Rightarrow \boxed{\Delta t = \frac{2.5\times10^8 m}{1.6(3\times10^8 m/s)} = .5208s}$

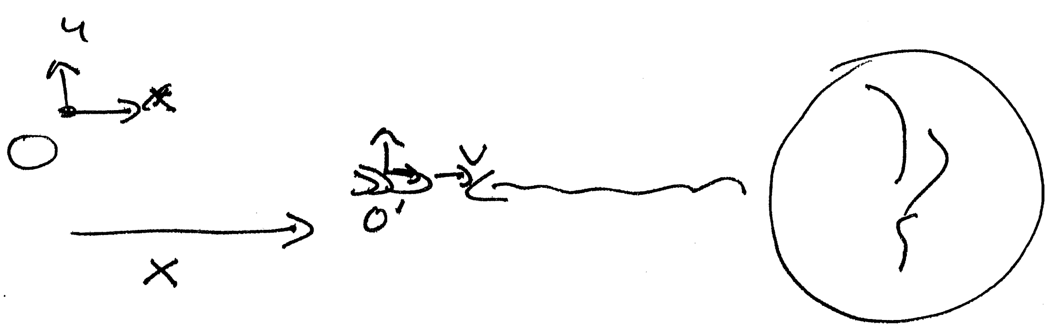
$\gamma = \frac{1}{\sqrt{1-.6^2}} = 1.25, \Rightarrow \boxed{\Delta t_0 = \frac{.5208s}{1.25} = .417s}$

Problem 1 Continued and Expanded (And I Knew this one was going to cause problems).

Go back to the definition of $t=0$ in Lorentz transform → This is time at which the two frames' origins co-incide. So our picture looks like:



When light meets spaceship



So light is emitted has spacetime co-ordinates: $X_1 = 2.5\times10^8\,m$, $t_1 = 0$

Light meets spaceship at $X_2 = .6c\,(.5208s) = .9375\times10^8\,m$

$t_2 = .5208s = \dfrac{2.5\times10^8\,m}{1.6\,(3\times10^8\,m/s)}$

Now Apply Lorentz: $x' = \gamma(x - vt)$, $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$\gamma = 1.25$ $\quad x_1 = 2.5\times10^8 m$, $t_1 = 0$

$\Rightarrow x_1' = 1.25(2.5\times10^8 m - 0) = 3.125\times10^8 m$

$$t_1' = 1.25\left(0 - \frac{.6c(2.5\times10^8 m)}{c(3\times10^8 m/s)}\right) = -.625s$$

$x_2 = .9375\times10^8 m$, $t_2 = .5208s = \frac{2.5\times10^8 m}{1.6(c)}$

$$\Rightarrow x_2' = 1.25\left(.9375\times10^8 m - .6c\left(\frac{2.5\times10^8 m}{1.6c}\right)\right) = 0$$ ← Because Light has reached spaceship which is location of '0' i.e. $x' = 0$

$$t_2' = 1.25\left(\frac{2.5\times10^8 m}{1.6(3\times10^8 m/s)} - \frac{.6c(.9375\times10^8 m)}{c(3\times10^8 m/s)}\right)$$

$= .417s$ ← INITIAL ANSWER ON PAGE 1

But .417s IS THE TIME SINCE $t' = 0$. $t' = 0$ is the time at which the two origins co-incide in rocket's frame. Imagine a great Big "O" floating in space, $2.5\times10^8 m$ Away from Earth. At $t = 0$ on Earth, spaceship flies past the giant "O" when light is emitted. But in rocket's frame, they pass the giant "O" After the light has been emitted; .625s After the light is emitted.

So in rocket's frame .417s after they pass the "0" the light reaches them. But to answer the original question, it has been .625s + .417s = 1.042s since the light was emitted when it reaches them.

Dang simultaneity! It get's me every time.

Now to go back to the question's original intent. When the light was emitted, the rocket is, in their frame, $x_1' = 3.125 \times 10^8$ m away from Earth.

Same speed of light in all frames, so we can also simply use $\frac{3.125 \times 10^8 \text{m}}{3 \times 10^8 \text{m/s}} = 1.042$s to find how long it takes light to reach the spaceship!

2. On October 15, 1991, the University of Utah's Fly's Eye cosmic ray detector measured a proton with total energy 51 J!!! (That's about the same energy as a baseball traveling at fifty-five miles per hour!!!) How fast was this proton going? Express your answer as $v = c(1 - \Delta)$ and solve for Δ. Use $1.67 \times 10^{-27}\ kg$ for the rest mass of a proton. Assume that Δ is small enough that $\Delta^2 \approx 0$.

$$E = 51J \Rightarrow E = \gamma M_0 c^2 = 51J$$

$$\Rightarrow \gamma = \frac{51J}{(1.67\times10^{-27}kg)(3\times10^8 m/s)^2} = \frac{51J}{1.503\times10^{-10}J} = 3.39\times10^{11}!$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \Rightarrow 1 - v^2/c^2 = \frac{1}{\gamma^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}$$

$$V = c(1-\Delta) \Rightarrow (1-\Delta)^2 = 1 - \frac{1}{\gamma^2} \Rightarrow 1 - 2\Delta + \cancel{\Delta^2}^{\approx 0} = 1 - \frac{1}{\gamma^2}$$

$$\Rightarrow 2\Delta = \frac{1}{\gamma^2} \Rightarrow \Delta = \frac{1}{2\gamma^2} = \frac{1}{2(3.39\times10^{11})^2} = 4.34\times10^{-24}$$

3. In collider experiments, particles are routinely accelerated to speeds extremely close to the speed of light and then smashed into each other. If two protons are heading towards each other with equal .999 c speeds (which is down-right pokey by today's standards), how fast is the second proton moving in the first proton's frame of reference?

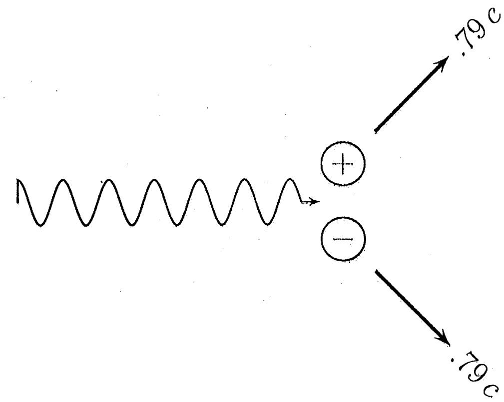
LAB FRAME PROTON FRAME

$\Rightarrow u = .999c, \quad V_x = -.999c$

In proton frame, $V_x' = ?$ $\quad V_x' = \dfrac{V_x - u}{1 - \frac{uV_x}{c^2}}$

$$V_x' = \frac{-.999c - .999c}{1 - \frac{(.999c)(-.999c)}{c^2}} = \frac{-1.998c}{1 + (.999)^2} = .9999995c$$

4. The positron is a particle that is identical in every way to an electron except that it has positive charge. Positrons are created when high energy light (~~energy~~) converts itself into a positron and electron pair. (One of each in order to conserve charge). How much energy must the light have in order to create a positron and electron, each with speed $.79c$? Use $9.11 \times 10^{-31}\ kg$ for the rest mass of the electron and positron.



To create electron with speed .79c

$\Rightarrow E_- = \gamma M_0 c^2 \leftarrow$ total Energy, $\gamma = \frac{1}{\sqrt{1-\frac{(.79c)^2}{c^2}}}$

to create positron $\Rightarrow E_+ = \gamma M_0 c^2$, $\leftarrow$ SAME γ AND M_0

$\Rightarrow E_{TOTAL} = 2\gamma M_0 c^2 = 2(1.631)(9.11\times10^{-31} kg)(3\times10^{8} m/s)^2$

$\boxed{= 2.67\times10^{-13} J}$

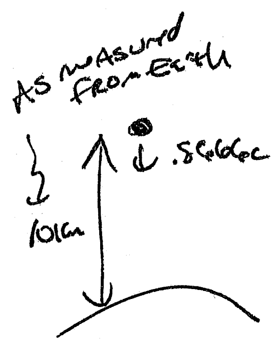
5. Muons are unstable subatomic particles that are produced when cosmic rays (mostly protons ejected from the sun) strike the Earth's upper atmosphere. They are a traditional example of the reality of special relativity because without it, no muons would ever reach the Earth's surface.

Muons are unstable in that they "decay", *i.e.*, they change into something else. In the case of Muons, they decay to electrons. The half-life of a muon at rest is $2.2\mu s$. The half-life tells us how long it takes for one half of a sample to decay. In other words if we started with 1000 at rest muons, after one half-life (after $2.2\,\mu s$) there would be 500 muons left (and 500 electrons). After two half-lives ($4.4\,\mu s$), there would be $250 = 1/2 * 500 = 1/2 * 1/2 * 1000$ muons left. In general, after l half-lives there would be

$$N = \left(\frac{1}{2^l}\right) 1000$$

muons left. (Note: l does not have to be an integer for this equation to work.)

Assume 1000 muons with $v = .8666\,c$ are created $10\,km$ above the Earth's surface. Predict how many muons will reach the Earth's surface. How many muons would reach the Earth's surface if special relativity was not "true"?



Earth measures dilated time

$$\Delta t = \frac{10000m}{.86666(3\times10^8 m/s)} = 3.846\times10^{-5}s$$

$$\gamma = \frac{1}{\sqrt{1-.86666^2}} = 2$$

$$\Rightarrow \Delta t_0 = 1.923\times10^{-5}s$$

IN MUON'S FRAME $l = \frac{1.923\times10^{-5}}{2.2\times10^{-6}} = 8.74 \Rightarrow N = \left(\frac{1}{2^{8.74}}\right)1000 = 2.33$

$\Rightarrow$ 2 still around when MUONS REACH EARTH

No relativity? $\Delta t_0 = \Delta t = 3.846\times10^{-5}s$

$\Rightarrow l = 17.5 \Rightarrow N = \left(\frac{1}{2^{17.5}}\right)1000 = .005$

$\Rightarrow$ NONE.