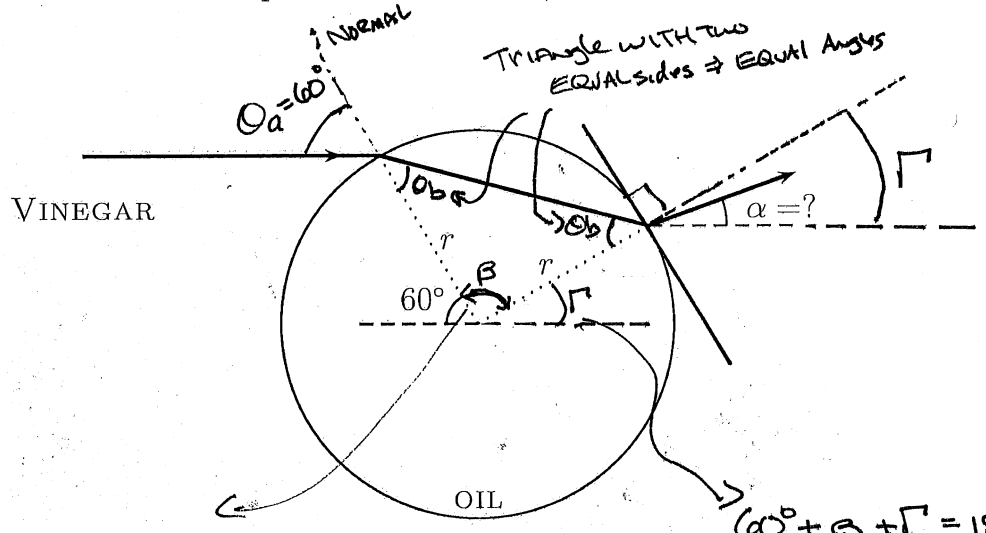
PHYSICS 262
EXAM 2

Please answer any four of the following five questions. Each question is worth five points. Partial credit will be awarded for any **attempted** problem.

1. In a colloid like mayonnaise or salad dressing, droplets of a fat are suspended in a liquid. In vinegar (index of refraction 1.37), horizontally propagating light enters a circular olive oil (index of refraction 1.47) droplet at 60°. At what angle α, relative to the horizontal, will the light leave the droplet? As a reminder, the normal lines of a circle are radial.



Σ Angles $= 180° \Rightarrow 2\theta_b + \beta = 180° \Rightarrow \beta = 180° - 2\theta_b$

$60° + \beta + \Gamma = 180°$

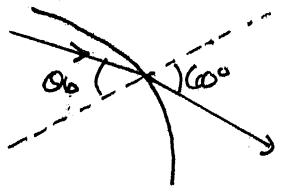
$\Rightarrow 60° + 180° - 2\theta_b + \Gamma = 180°$

$\Rightarrow \Gamma = 2\theta_b - 60°$

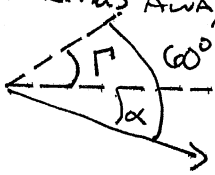
Snell's Law: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow 1.37 \sin 60° = 1.47 \sin\theta_b$

$\Rightarrow \theta_b = 53.8° \Rightarrow \Gamma = 47.6°$

At other side of droplet, light hits at θ_b again ⇒ refracts out at 60° again



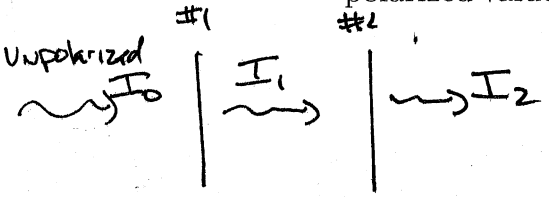
Sorry, it was mean but I drew α in the wrong direction to see if you would catch it. Remember light bends away from normal going slow to fast. Real picture:



$\alpha = 60° - \Gamma = 60° - 47.6° = 12.4°$

OR better yet, $\alpha = -12.4°$

2. Unpolarized light passes through a polarizer whose transmission axis is at 45°. After passing through another polarizer of unknown transmission axis, the light's intensity is 0.49 of its unpolarized value. If the first polarizer's transmission axis is increased to 75°, the light's intensity, after passing through the second polarizer, is 0.31 of its unpolarized value. What angle is the second polarizer's transmission axis?



Unpolarized $\Rightarrow I_1 = \frac{1}{2} I_0$ AND light polarized at θ_1

LAW OF MALUS $\Rightarrow I_2 = I_1 \cos^2(\theta_1 - \theta_2) = \frac{1}{2} I_0 \cos^2(\theta_1 - \theta_2)$

(θ_2: Transmission Axis of #2)

WHEN $\theta_1 = 45°$, $I_2 = .49 I_0 \Rightarrow .49 I_0 = \frac{1}{2} I_0 \cos^2(45° - \theta_2) \Rightarrow \cos^2(45° - \theta_2) = .98$

$\Rightarrow \cos(45° - \theta_2) = \sqrt{.98}$. COSINE IS EVEN $\Rightarrow 45° - \theta_2 = \pm \cos^{-1}(\sqrt{.98}) = \pm 8.13°$

$\Rightarrow \theta_2 = 53.13°$ OR $36.87°$.

WHEN $\theta = 75°$, $.31 = \frac{1}{2} \cos^2(75° - \theta_1) \Rightarrow \theta_1 = 75° \pm \cos^{-1}(\sqrt{.62}) = 113°$ OR $36.90°$ (ROUNDING)

$\Rightarrow \boxed{\theta_2 = 36.90°}$

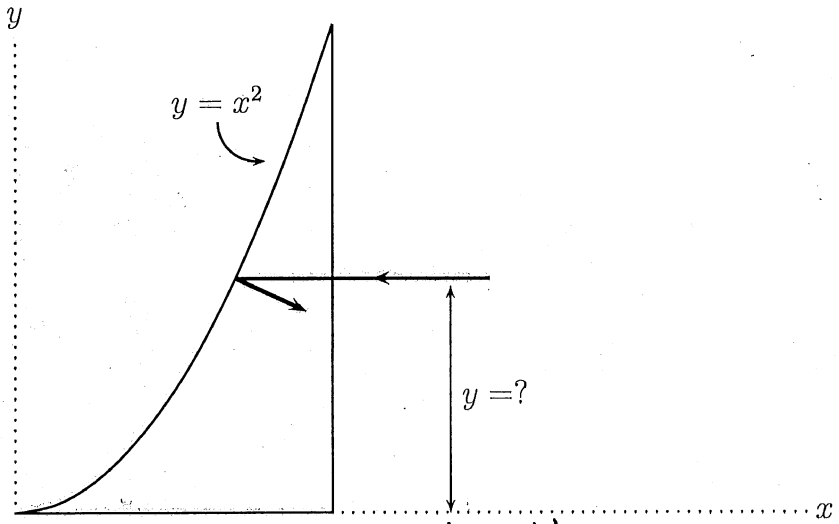
3. A beam of light inside of alcohol (index of refraction 1.36) strikes ice (whose index is 1.31, slightly different than liquid water). At what angle will the light reflected from the ice's surface be totally polarized?

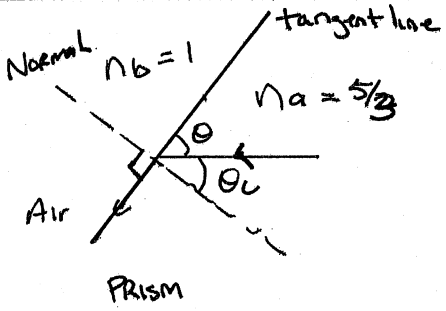
BREWSTER'S ANGLE: $n_a \tan \theta_p = n_b$ (a: ALCOHOL, b: ICE)

$\Rightarrow 1.36 \tan \theta_p = 1.31$

$\Rightarrow \boxed{\theta_p = \tan^{-1}\left(\frac{1.31}{1.36}\right) = 43.9°}$

4. A beam of light in air enters a parabolic prism (index of refraction 5/3) as shown. If the beam totally internally reflects off the inside surface of the prism, what is the ~~minimum~~ value of the beam's height, y? As a reminder, the angle θ of the tangent line of a curve is given by $\tan\theta = \frac{dy}{dx}$.





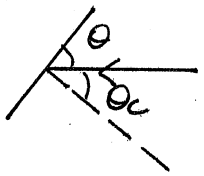
CRITICAL ANGLE:

$n_a \sin\theta_a = n_b \sin\theta_b$

$\Rightarrow \frac{5}{3}\sin\theta_c = (1)\sin 90°$

$\Rightarrow \sin\theta_c = \frac{3}{5}$

θ = Angle of tangent line

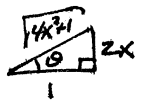


$\theta + \theta_c = 90° \Rightarrow \theta_c = 90° - \theta$

$\Rightarrow \sin(90° - \theta) = \frac{3}{5}$

$\Rightarrow \cos\theta = 3/5$

$\tan\theta = \frac{dy}{dx}$ $\quad y = x^2 \Rightarrow \tan\theta = 2x$



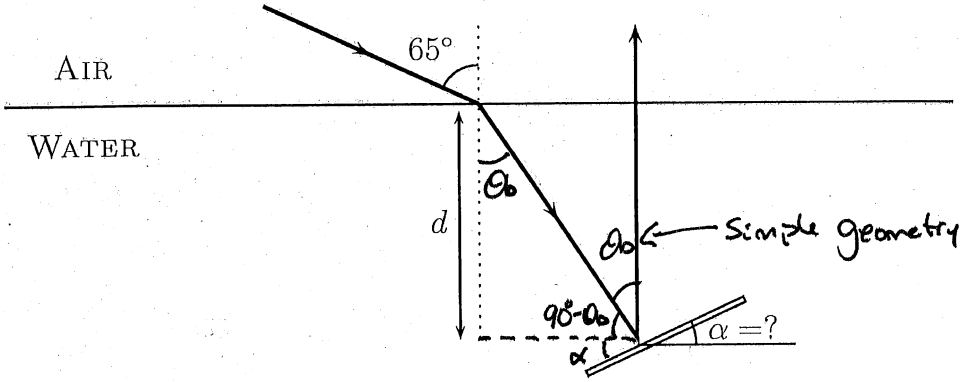
$\Rightarrow \cos\theta = \frac{1}{\sqrt{4x^2+1}}$

(NOTE: $\cos\theta = \frac{1}{\sqrt{4x^2+1}} \Rightarrow \theta$ decreases as x decreases $\Rightarrow 90° - \theta$ increases as x decreases $\Rightarrow 90° - \theta$ increases as y ~~decreases~~ increases $\Rightarrow$ we'll find maximum y.)

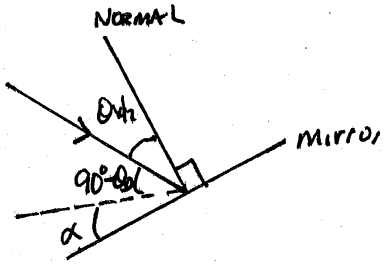
$\frac{1}{\sqrt{4x^2+1}} = \frac{3}{5} \Rightarrow \frac{1}{4x^2+1} = \frac{9}{25} \Rightarrow \frac{25}{9} = 4x^2+1 \Rightarrow 4x^2 = \frac{25}{9} - 1 = \frac{16}{9}$

$\Rightarrow x^2 = \frac{4}{9}$ $\quad y = x^2 \Rightarrow y = 4/9 = .444$

5. From air, light enters water (index of refraction 4/3) at an angle of 65°. (As shown the boundary between the air and water is horizontal.) A vertical distance d below the surface there is a plane mirror. At what angle α should the mirror be placed so as to cause the light reflected off the mirror to travel upwards and hit the surface at 0°?



LAW OF REFLECTION $\Rightarrow \theta_i = \theta_r = \theta_b/2$



$$\alpha + (90° - \theta_b) + \frac{\theta_b}{2} = 90°$$

$$\Rightarrow \alpha - \theta_b + \frac{\theta_b}{2} = 0$$

$$\Rightarrow \alpha - \frac{\theta_b}{2} = 0 \Rightarrow \alpha = \theta_b/2$$

LAW OF REFRACTION: $n_a \sin\theta_a = n_b \sin\theta_b$

$$\Rightarrow (1)\sin 65° = (4/3)\sin\theta_b \Rightarrow \theta_b = \sin^{-1}\left(\tfrac{3}{4}\sin 65°\right) = 42.8°$$

$$\Rightarrow \boxed{\alpha = \frac{42.8°}{2} = 21.4°}$$