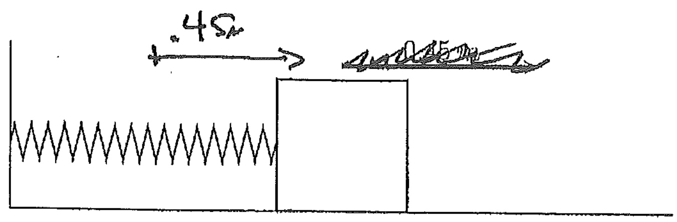
1. A 2 kg mass is attached to a 50 N/m spring as shown below. The mass is pulled 0.45 m to the right of its equilibrium position and then released from rest. There is no friction between the mass and the floor.



$X_o = .45m$

$V_o = 0$

$SHM \Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50N/m}{2kg}}$
$= 5 rad/s$

(a) What is the phase angle, ϕ, in the equation $x = A\cos(\omega t + \phi)$ for this motion?

(b) What is the amplitude of this motion?

(c) What is the period for this motion?

$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi rad}{5 rad/s}$

$\Rightarrow \boxed{T = 1.26s}$

$X = A\cos(\omega t + \phi) \quad V = -\omega A \sin(\omega t + \phi)$

$X_o = X(t=0) = A\cos\phi, \quad V_o = V(t=0) = -\omega A \sin\phi$

$\therefore .45m = A\cos\phi, \quad 0 = -5A\sin\phi \leftarrow \sin\phi = 0$
$\Rightarrow \phi = 0$

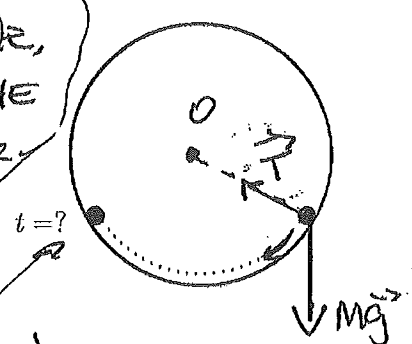
$\boxed{\phi = 0 rad} \Rightarrow A\cos 0 = .45m \Rightarrow \boxed{A = 0.45m}$

2. A periodic wave has a speed of $10\,m/s$ and $5\,m$ wavelength. What is the wave's period?

$$V = f\lambda = \frac{\lambda}{T} \Rightarrow T = \frac{\lambda}{V} = \frac{5m}{10m/s} = \frac{1}{2}s = 0.5s$$

3. A small mass is started from rest one-third of the way up a frictionless circle of radius $2.45\,m$. How long does it take for the mass to slide to the other side? Assume this motion occurred on Earth. **HINT:** Compare the forces acting on this mass to the ones acting on the simple pendulum.

NOTE: PERIOD IS INDEPEDENT OF Amplitude, SO THE TIME would be THE SAME FOR ANY point where SMALL-ANGLE APPROXIMATION holds



TO THIS point IS HALF OF A CYCLE

$\vec{T}$ CAUSES NO TORQUE $\Rightarrow$ SAME EQUATION AS Simple pendulum

Assuming $\frac{1}{3}$ of THE WAY up, SMALL enough FOR SMALL Angle Approximation

$$\omega = \sqrt{\frac{g}{L}} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/L}} = 2\pi\sqrt{\frac{L}{g}}$$

HERE, $L = r = 2.45m$

$$\Rightarrow T = 2\pi\sqrt{\frac{2.45m}{9.8m/s^2}} = 2\pi\sqrt{\frac{1}{4}s^2} = 2\pi\left(\frac{1}{2}s\right) = \pi s = 3.14s$$

to go HALF A cycle $t = \frac{T}{2} = \frac{\pi s}{2} = (\pi/2)s = 1.57s$

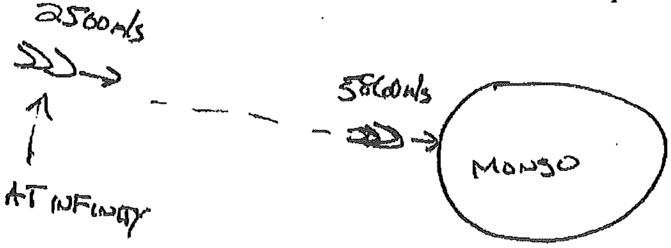
4. Through circumstances too bizarre to be detailed here, you find yourself marooned on an extrasolar, Earth-like planet!! In "honor" of Mastering Physics (whose psychological scars remain with you wherever you go), you christen your new home Planet Mongo.

   (a) Taking your Physics instructor's always excellent advice, you immediately measure the period of a simple pendulum. If you find that a 0.35 m long pendulum has a 1.57 s period, what is the acceleration due to gravity on Mongo?

Simple pendulum, $\omega = \sqrt{\frac{g}{L}} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/L}} = 2\pi\sqrt{\frac{L}{g}}$

$\therefore T^2 = \frac{4\pi^2 L}{g} \Rightarrow g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(.35m)}{(1.57s)^2} = 5.61\,m/s^2$

(b) From data recovered from your wrecked spaceship, you determine that your vehicle's speed was 5860 m/s when it entered Mongo's atmosphere (and was effectively a distance of one radius from Mongo's center). When your engines stopped working, your spaceship was traveling at 2500 m/s. Assuming you were infinitely far away from Mongo at this point (when gravity became the only force doing work on your spaceship), determine the mass and radius of your new home planet. HINT: You will need to use your result from part (a).



CONSERVATION OF ENERGY (GRAVITY ONLY FORCE DOING WORK)

$$\frac{1}{2}mV_1^2 - \frac{GmM_M}{r_1} = \frac{1}{2}mV_2^2 - \frac{GmM_M}{r_2}$$

m = spaceship, M_M = Mongo's MASS

$V_1 = 2500\,m/s$, $r_1 \to \infty$, $V_2 = 5860\,m/s$, $r_2 = R_M$

Mongo's RADIUS

$$\Rightarrow \frac{1}{2}V_1^2 = \frac{1}{2}V_2^2 - \frac{GM_M}{R_M} \Rightarrow \frac{GM_M}{R_M} = \frac{1}{2}(V_2^2 - V_1^2) = \frac{1}{2}\left[(5860\,m/s)^2 - (2500\,m/s)^2\right]$$

$$\Rightarrow \frac{GM_M}{R_M} = 1.40448 \times 10^7\,m^2/s^2.$$ From part (a) we know $g = 5.61\,m/s^2$

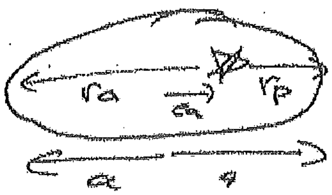
ON ANY planet $g = \frac{GM_M}{R_M^2} \Rightarrow \frac{GM_M}{R_M} = gR_M \Rightarrow gR_M = 1.40448\times10^7\,m^2/s^2$

$$\Rightarrow \boxed{R_M = \frac{1.40448\times10^7\,m^2/s^2}{5.61\,m/s^2} = 2.5\times10^6\,m}$$

$$M_M = \frac{(1.404\times10^7\,m^2/s^2)R_M}{G} = 5.26\times10^{23}\,Kg$$

5. The most famous of comets is, of course, Halley's comet (mass $2.2 \times 10^{14}\, kg$). It orbits the sun (mass $1.99 \times 10^{30}\, kg$) with a period of $75.3\, years$ on a highly elliptical orbit of eccentricity 0.967.

(a) Find the closest and farthest distance between the sun and Halley's comet, *i.e*, find the perihelion and aphelion distances.



First find a from $T = \frac{2\pi a^{3/2}}{\sqrt{GM_s}}$ ↳ sun's mass

$$\Rightarrow T^2 = \frac{4\pi^2 a^3}{GM_s} \Rightarrow a^3 = \frac{T^2 GM_s}{4\pi^2}$$

$$T = 75.3\,\text{year} \times \frac{365\,\text{day}}{\text{year}} \times \frac{24\,h}{\text{day}} \times \frac{3600\,s}{h} = 2.375\times10^{9}\,s$$

$$a^3 = \frac{(2.375\times10^{9}\,s)^2 (6.67\times10^{-11}\,N\cdot m^2/kg^2)(1.99\times10^{30}\,kg)}{4\pi^2} = 1.9\times10^{37}\,m^3$$

$$\Rightarrow a = 2.67\times10^{12}\,m$$

$$ea + r_p = a \Rightarrow r_p = a(1-e) = 2.67\times10^{12}\,m\,(1-.967)$$

$$= 2.67\times10^{12}\,m\,(.033)$$

$$= 8.8\times10^{10}\,m$$

$$r_a = a + ea = a(1+e) = 2.67\times10^{12}\,m\,(1+.967) = 2.67\times10^{12}\,m\,(1.967)$$

$$= 5.25\times10^{12}\,m$$

(b) Find the speed of Halley's comet at perihelion and aphelion.

Like Homework problem! AT perihelion, $L = mV_p r_p$

At Aphelion, $L = mV_a r_a$. Conservation

$\Rightarrow V_p r_p = V_a r_a \Rightarrow V_p = V_a \left(\frac{r_a}{r_p}\right) = V_a \frac{a(1+e)}{a(1-e)}$

$\Rightarrow V_p = V_a \left(\frac{1.967}{.033}\right) = 59.6\, V_a$

Energy: $\frac{1}{2} \cancel{m} V_a^2 - \frac{G\cancel{m}M_s}{r_a} = \frac{1}{2}\cancel{m} V_p^2 - \frac{G\cancel{m}M_s}{r_p}$

$\Rightarrow GM_s\left(\frac{1}{r_p} - \frac{1}{r_a}\right) = \frac{1}{2}(V_p^2 - V_a^2) \Rightarrow 2GM_s\left(\frac{1}{r_p} - \frac{1}{r_a}\right) = V_p^2 - V_a^2$

$\Rightarrow (59.6V_a)^2 - V_a^2 = [59.6^2 - 1]V_a^2 = 2GM_s\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$

$\Rightarrow 3551.16\, V_a^2 = 2(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{1}{8.8\times10^{10}} - \frac{1}{5.25\times10^{12}}\right)$

$\Rightarrow 3551.16\, V_a^2 = 2(6.67\times10^{-11})(1.99\times10^{30})(1.117\times10^{-11})$

$\Rightarrow V_a^2 = \frac{2.966\times10^{9}}{3551.16} \Rightarrow \boxed{V_a = 913.9 \cong 914\,m/s}$

$\boxed{V_p = 59.6V_a = 54\,500\,m/s}$