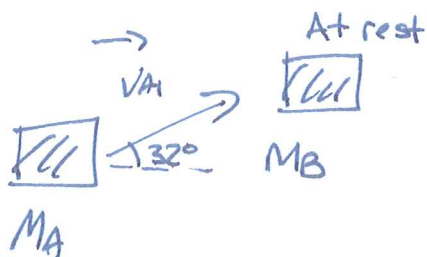


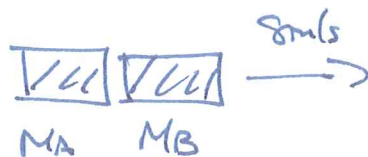
PRACTICE EXAM 4

- (1.) A $M_A = 5 \text{ kg}$ mass with $\vec{v}_{A1} = 25 \text{ m/s}$ at 32° (relative to the $+x$ -axis) collides with a stationary $M_B = 15 \text{ kg}$ mass. If M_B bounces to the right with a speed of 8 m/s , with what speed does M_A bounce?

AND Direction



Before



After

Must Conserve Momentum. $M_A v_{A1,x} + M_B v_{B1,x} = M_A v_{A2,x} + M_B v_{B2,x}$
 $M_A v_{A1,y} + M_B v_{B1,y} = M_A v_{A2,y} + M_B v_{B2,y}$

$$v_{A1,x} = 25 \text{ m/s} \cos 32^\circ, \quad v_{B1,x} = 0, \quad v_{A2,x} = ?, \quad v_{B2,x} = 8 \text{ m/s}$$

$$v_{A1,y} = 25 \text{ m/s} \sin 32^\circ, \quad v_{B1,y} = 0, \quad v_{A2,y} = ?, \quad v_{B2,y} = 0$$

$$\Rightarrow 5 \text{ kg} (25 \text{ m/s}) \cos 32^\circ + 0 = 5 \text{ kg} v_{A2,x} + 15 \text{ kg} (8 \text{ m/s})$$

$$\Rightarrow 106 \text{ kg} \cdot \text{m/s} = 5 \text{ kg} v_{A2,x} + 120 \text{ kg} \cdot \text{m/s} \Rightarrow v_{A2,x} = \frac{106 \text{ kg} \cdot \text{m/s} - 120 \text{ kg} \cdot \text{m/s}}{5 \text{ kg}}$$

$$= \frac{-14 \text{ kg} \cdot \text{m/s}}{5 \text{ kg}} = -2.8 \text{ m/s}$$

x-Component:

$$5\text{kg}(25\text{m/s})\sin 32^\circ + 0 = 5\text{kg} V_{A2,y} + 0$$

$$\Rightarrow V_{A2,y} = 25\text{m/s} \sin 32^\circ = 13.25\text{m/s}$$

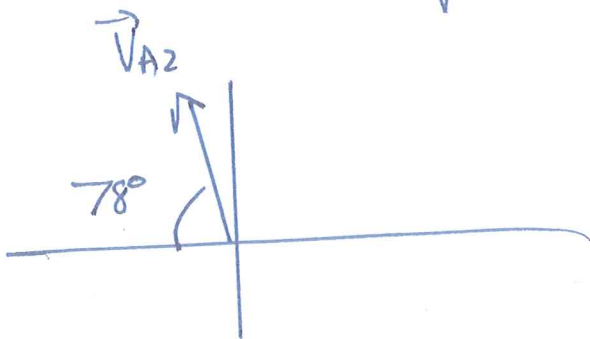
$$V_{A2} = \sqrt{V_{A2,x}^2 + V_{A2,y}^2} = \sqrt{(2.8\text{m/s})^2 + (13.25\text{m/s})^2} = 13.54\text{m/s} \\ = 13.5\text{m/s}$$

$V_{A2,x} < 0, V_{A2,y} > 0 \Rightarrow 2^{\text{ND}}$ QUADRANT

SO EITHER FIND STANDARD ANGLE: $\theta = \tan^{-1}\left(\frac{V_{A2,y}}{V_{A2,x}}\right) + 180^\circ$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{13.25}{-2.8}\right) + 180^\circ = -78^\circ + 180^\circ = 102^\circ$$

OR ~~USE~~ DRAW ME A PICTURE



- 2
2. A $M_A = 5 \text{ kg}$ mass with $v_{A1} = 6 \text{ m/s}$ and a $M_B = 7 \text{ kg}$ mass with $v_{B1} = 2 \text{ m/s}$ have an elastic collision. If $v_{A2} = 3 \text{ m/s}$, v_{B2} has what value?

Since we know collision is elastic:

$$\frac{1}{2} M_A v_{A1}^2 + \frac{1}{2} M_B v_{B1}^2 = \frac{1}{2} M_A v_{A2}^2 + \frac{1}{2} M_B v_{B2}^2$$

$$\Rightarrow \frac{1}{2} (5 \text{ kg}) (6 \text{ m/s})^2 + \frac{1}{2} (7 \text{ kg}) (2 \text{ m/s})^2 = \frac{1}{2} (5 \text{ kg}) v_{A2}^2 + \frac{1}{2} (7 \text{ kg}) (3 \text{ m/s})^2$$

$$\Rightarrow 90 \text{ J} + 14 \text{ J} = \frac{1}{2} (5 \text{ kg}) v_{A2}^2 + 31.5 \text{ J}$$

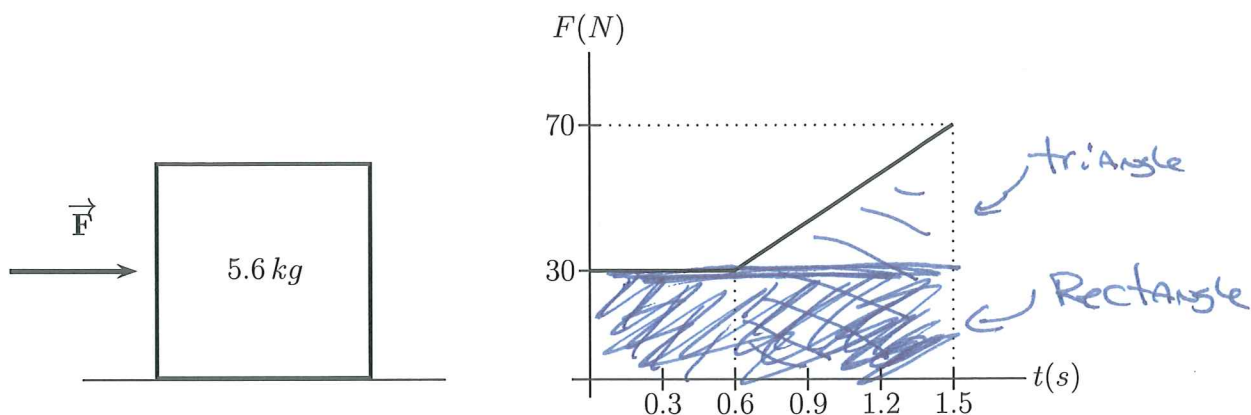
$$\Rightarrow 104 \text{ J} = \frac{1}{2} (5 \text{ kg}) v_{A2}^2 + 31.5 \text{ J}$$

$$\Rightarrow v_{A2} = \sqrt{\frac{2(104 \text{ J} - 31.5 \text{ J})}{5 \text{ kg}}} = \sqrt{\frac{2(72.5 \text{ J})}{5 \text{ kg}}}$$

$$= 5.39 \text{ m/s}$$

3

- (3) A 5.6 kg box is sitting stationary on a frictionless surface when your instructor steps up to it and applies, in the positive x direction, a non-constant force, F .



- (a.) What impulse was imparted to the mass by your instructor's force?
 (b.) What was the average force exerted on the mass?
 (c.) How fast was the 5.6 kg , initially at rest, mass going after 1.5 s ?

a. Variable Force $\Rightarrow J = \text{Area} = (1.5\text{s})(30\text{N}) + \frac{1}{2}(1.5\text{s}-0.6\text{s})(70\text{N}-30\text{N})$

$$\Rightarrow J = (1.5\text{s})(30\text{N}) + \frac{1}{2}(0.9\text{s})(40\text{N}) = 45\text{N}\cdot\text{s} + 18\text{N}\cdot\text{s}$$

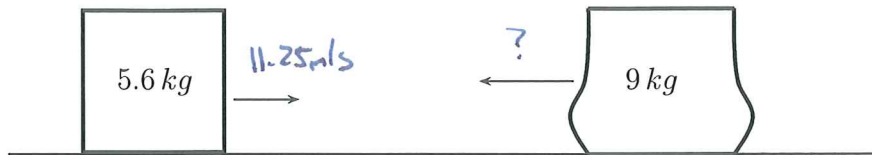
$$= 63\text{N}\cdot\text{s} = 63\text{kg}\cdot\text{m/s}$$

b. $J = F_{\text{AV}} \Delta t \Rightarrow F_{\text{AV}} = \frac{J}{\Delta t} = \frac{63\text{N}\cdot\text{s}}{1.5\text{s}} = 42\text{N}$

c.) $J = \Delta p$ since \vec{F} in x -direction only $J_x = \Delta p_x$

$$\Rightarrow J_x = mV_{2,x} - mV_{1,x}^0 \Rightarrow V_{2,x} = \frac{J_x}{m} = \frac{63\text{kg}\cdot\text{m/s}}{5.6\text{kg}} = 11.25\text{m/s}$$

5. (6.) After being pushed, the 5.6 kg box slides across the frictionless floor and collides with a 9.0 kg lump of clay that is traveling in the opposite direction. The box and the clay stick to each other and have a post-collision velocity of 0.5 m/s to the right.



- (a.) How fast was the clay going the instant before the collision?
 (b.) How much heat was produced during this collision?

Completely Inelastic, only in x-direction

$$\Rightarrow M_A V_{A1} + M_B V_{B1} = (M_A + M_B) V_2$$

$$V_{A1} = 11.25 \text{ m/s}, V_{B1} = ?, V_2 = 0.5 \text{ m/s}$$

$$\Rightarrow (5.6 \text{ kg})(11.25 \text{ m/s}) + (9 \text{ kg}) V_{B1} = (5.6 \text{ kg} + 9 \text{ kg})(0.5 \text{ m/s})$$

$$\Rightarrow 63 \text{ kg} \cdot \text{m/s} + (9 \text{ kg}) V_{B1} = (14.6 \text{ kg})(0.5 \text{ m/s}) = 7.3 \text{ kg} \cdot \text{m/s}$$

$$\Rightarrow (9 \text{ kg}) V_{B1} = 7.3 \text{ kg} \cdot \text{m/s} - 63 \text{ kg} \cdot \text{m/s} = -55.7 \text{ kg} \cdot \text{m/s} \Rightarrow V_{B1} = \frac{-55.7 \text{ kg} \cdot \text{m/s}}{9 \text{ kg}} = -6.189 \text{ m/s}$$

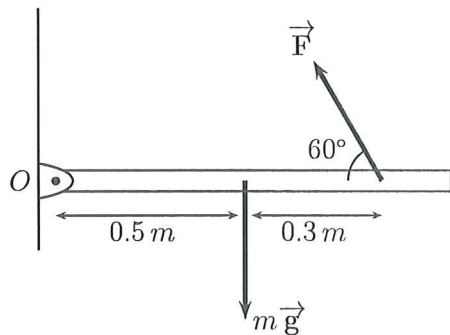
↑
left

b.) Missing Kinetic Energy change into heat

$$K_1 = \frac{1}{2} M_A V_{A1}^2 + \frac{1}{2} M_B V_{B1}^2 = \frac{1}{2} (5.6 \text{ kg})(11.25 \text{ m/s})^2 + \frac{1}{2} (9 \text{ kg})(6.189 \text{ m/s})^2 = 526.7 \text{ J}$$

$$K_2 = \frac{1}{2} (M_A + M_B) V_2^2 = \frac{1}{2} (14.6 \text{ kg})(0.5 \text{ m/s})^2 = 1.825 \text{ J} \Rightarrow 524.875 \text{ J} = 525 \text{ J of heat}$$

- Newton's 2nd Law*
1. A ~~uniform~~ *uniform* 6.3 kg, 1 m long slender rod is attached to a vertical wall and can pivot about the point O shown below. The rod is being held horizontal by an applied force \vec{F} . What is the magnitude of the force \vec{F} shown above if the rod does not rotate? What are the directions of the torque exerted by \vec{F} and the torque exerted by gravity?



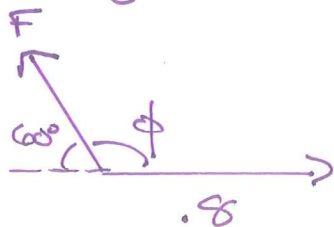
No Rotation $\Rightarrow \sum \vec{\tau} = 0 \Rightarrow \vec{\tau}_g + \vec{\tau}_F = 0$

\uparrow GRAVITY torque \uparrow TORQUE OF \vec{F}

From RHR, $\vec{\tau}_g = \otimes$, $\vec{\tau}_F = \odot \Rightarrow \vec{\tau}_g + \vec{\tau}_F = 0$

LS $\tau_g - \tau_F = 0 \Rightarrow \tau_g = \tau_F$

$\tau_g = X Mg = (0.5 \text{ m})(6.3 \text{ kg})(9.8 \text{ m/s}^2) = 30.87 \text{ N}\cdot\text{m}$

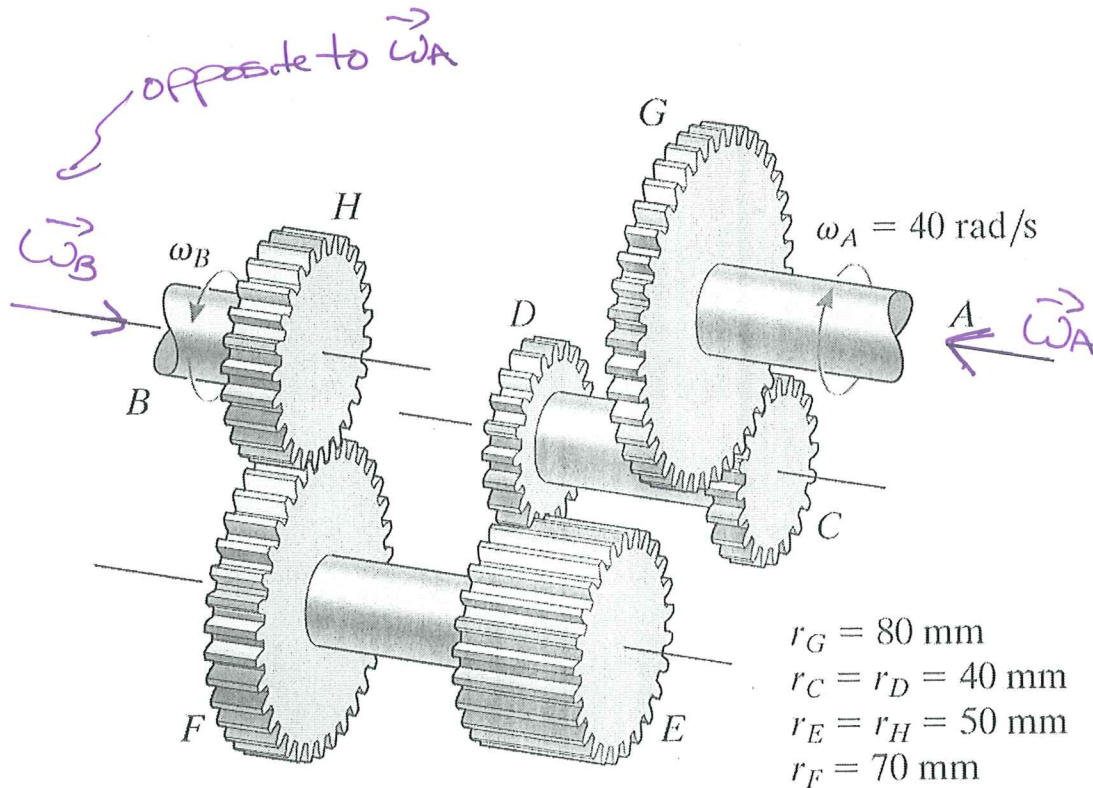


$\tau_F = (0.8 \text{ m}) F \sin 120^\circ = F(0.69282 \text{ m})$

$\therefore 30.87 \text{ N}\cdot\text{m} = F(0.69282 \text{ m})$

$\Rightarrow \boxed{F = 44.557 \text{ N} = 44.6 \text{ N}}$

2. The reverse gear of a car is made possible through a clever arrangement of gears and axles as schematically shown below. If the radius of each gear is as given in the figure and the car's engine rotates gear A with $\omega_A = 40 \text{ rad/s}$, what is the angular velocity of the the "drive shaft", B?



A & G, C & D, E & F, H & B ^{EACH} ON SAME AXLE \Rightarrow SAME Angular Velocity $\Rightarrow \omega_A = \omega_G, \omega_C = \omega_D, \omega_E = \omega_F, \omega_B = \omega_H$

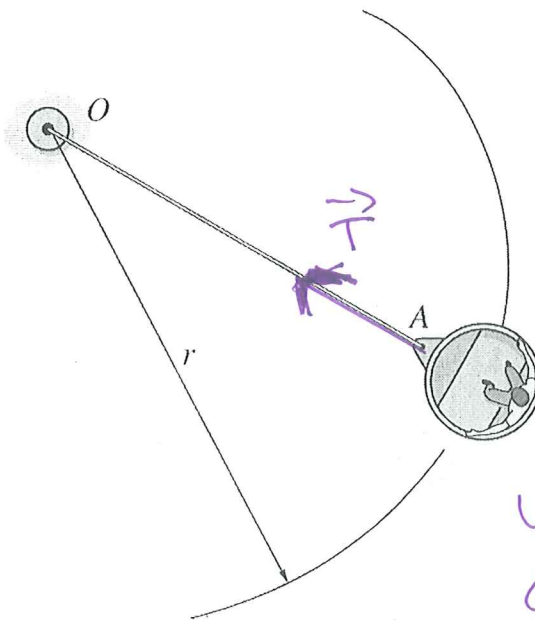
G & C, D & E, F & H IN CONTACT \Rightarrow SAME LINEAR VELOCITIES AT EDGE, $\Rightarrow V = \omega r \Rightarrow \omega_G r_G = \omega_C r_C, \omega_D r_D = \omega_E r_E, \omega_F r_F = \omega_H r_H$

$$\omega_G = \omega_A = 40 \text{ rad/s} \Rightarrow \omega_C = 40 \text{ rad/s} \left(\frac{80}{40} \right) = 80 \text{ rad/s} \Rightarrow \omega_D = 80 \text{ rad/s}$$

$$\omega_E = 80 \text{ rad/s} \left(\frac{40}{50} \right) = 64 \text{ rad/s}, \Rightarrow \omega_F = 64 \text{ rad/s},$$

$$\omega_H = 64 \text{ rad/s} \left(\frac{70}{50} \right) = 89.6 \text{ rad/s} \quad \therefore \boxed{\omega_B = 89.6 \text{ rad/s}}$$

3. An amusement park ride has a car, initial speed 6 m/s and distance $r = 4\text{ m}$, traveling in a horizontal circular path. If the distance r is slowly decreased (so the car always goes around in a circle) to 1.5 m , how fast will the car be going? Ignore friction.



No Friction so
Tension, Weight,
NORMAL.

Weight = \otimes

NORMAL = \ominus

Weight, NORMAL
CANCEL

\vec{T} EXERTS NO TORQUE

SINCE 180° between \vec{r} AND \vec{T}

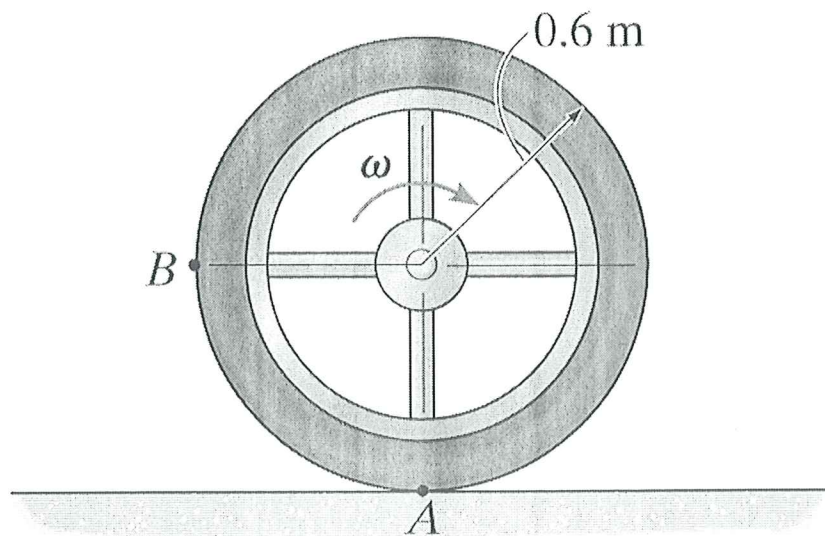


THEREFORE NO NET TORQUE \Rightarrow CONSERVATION OF
ANGULAR MOMENTUM. ALWAYS IN CIRCLE $\Rightarrow MV_1r_1 = MV_2r_2$

$$\therefore V_1r_1 = V_2r_2 \Rightarrow (6\text{ m/s})(4\text{ m}) = V_2(1.5\text{ m})$$

$$\Rightarrow \underline{\underline{V_2 = 16\text{ m/s}}}$$

4. The 9-kg wheel shown below, when its center is given an initial speed of 4.3 m/s, rolls without slipping up a hill to a height of 6 m. What is the wheel's moment of inertia?



$$\text{Rolls without slipping} \Rightarrow \frac{1}{2} M V_1^2 \left(1 + \frac{I}{MR^2}\right) + mgy_1 = \frac{1}{2} M V_2^2 \left(1 + \frac{I}{MR^2}\right) + mgy_2$$

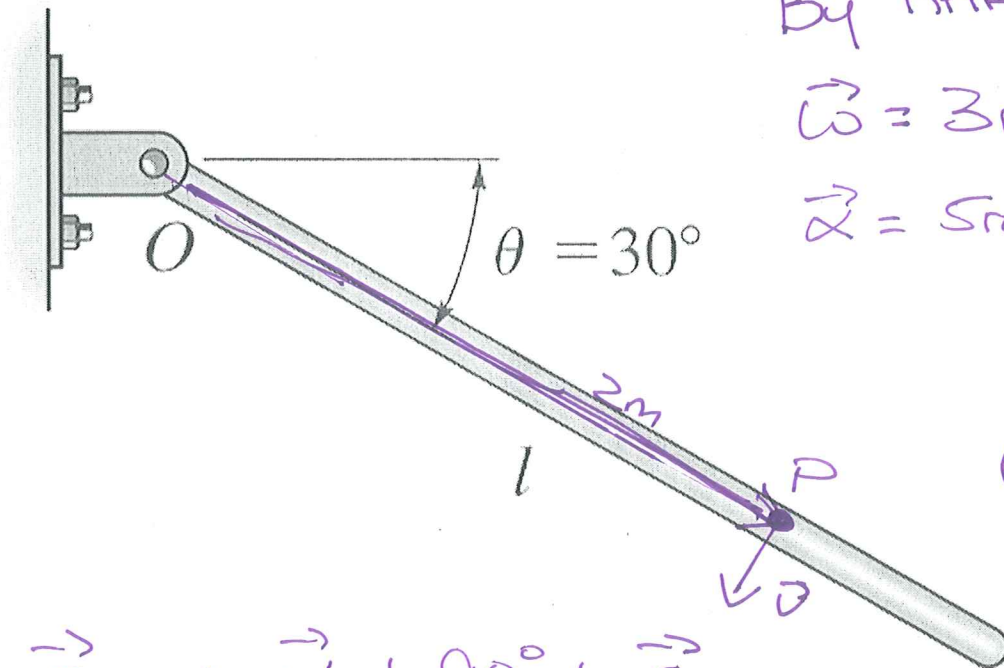
$$V_1 = 4.3 \text{ m/s}, y_1 = 0, V_2 = 0, y_2 = 6 \text{ m}$$

$$\Rightarrow \frac{1}{2} M V_1^2 \left(1 + \frac{I}{MR^2}\right) = mgy_2 \Rightarrow 1 + \frac{I}{MR^2} = \frac{2gy_2}{V_1^2} = \frac{2(9.8 \text{ m/s}^2)(6 \text{ m})}{(4.3 \text{ m/s})^2}$$

$$\Rightarrow 1 + \frac{I}{MR^2} = 6.36 \therefore I = MR^2(5.36) = (9 \text{ kg})(.6 \text{ m})^2(5.36)$$

$$\Rightarrow I = 17.367 \text{ kg} \cdot \text{m}^2$$

5. The $l = 2.5\text{-m}$ long stick has, at the instant shown, angular speed $\omega = 3\text{ rad/s}$ and angular velocity $\alpha = 5\text{ rad/s}^2$ both in the clockwise sense. What is the magnitude and direction of the angular velocity, the angular acceleration, the linear velocity, and the linear acceleration of the point at a distance 2 m from O ?

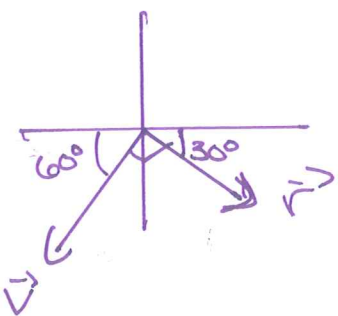


By RHR

$$\vec{\omega} = 3\text{ rad/s}, (\otimes)$$

$$\vec{\alpha} = 5\text{ rad/s}^2 (\otimes)$$

$$\vec{V} = \vec{\omega} \times \vec{r} \Rightarrow \vec{V} \text{ at } 90^\circ \text{ to } \vec{r}$$



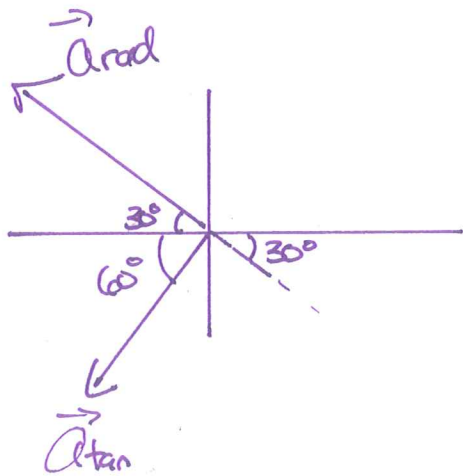
$$V = \omega r = (3\text{ rad/s})(2\text{ m}) = 6\text{ m/s}$$

$$\vec{a}_{\text{tan}} = \vec{\alpha} \times \vec{r} \Rightarrow \vec{a}_{\text{tan}} \text{ IN SAME DIRECTION AS } \vec{V}$$

$$a_{\text{tan}} = \alpha r = (5\text{ rad/s}^2)(2\text{ m}) = 10\text{ m/s}^2$$

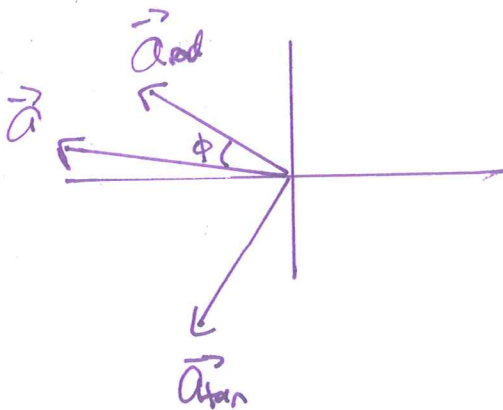
$$\vec{a}_{\text{rad}} = -\omega^2 \vec{r} \Rightarrow \text{opposite to } \vec{r}$$

$$a_{\text{rad}} = \omega^2 r = (3 \text{ rad/s})^2 (2 \text{ m}) = 18 \text{ m/s}^2$$



$$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{(18 \text{ m/s}^2)^2 + (10 \text{ m/s}^2)^2}$$

$$\Rightarrow a = 20.59 \text{ m/s}^2$$



$$\tan \phi = \frac{a_{\text{tan}}}{a_{\text{rad}}} \Rightarrow \phi = \tan^{-1}\left(\frac{10}{18}\right)$$

$$\Rightarrow \phi = 29^\circ$$

\vec{a}_{rad} at 30° above $-x$ -axis

$\Rightarrow \vec{a}$ is 1° above $-x$ -axis

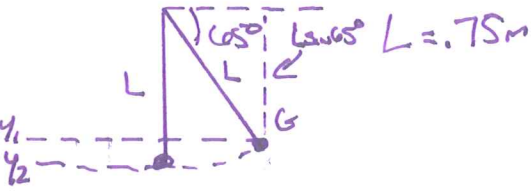
6. The 12-kg "physical-pendulum" with moment of inertia $5.5 \text{ kg} \cdot \text{m}^2$ and center of mass at G is released from rest at $\theta = 65^\circ$. How fast will it be going when it swings through the vertical ($\theta = 90^\circ$)?

CONSERVATION OF ENERGY:

$$\frac{1}{2} I \omega_1^2 + Mgy_1 = \frac{1}{2} I \omega_2^2 + Mgy_2$$

$$\omega_1 = 0, \omega_2 = ?$$

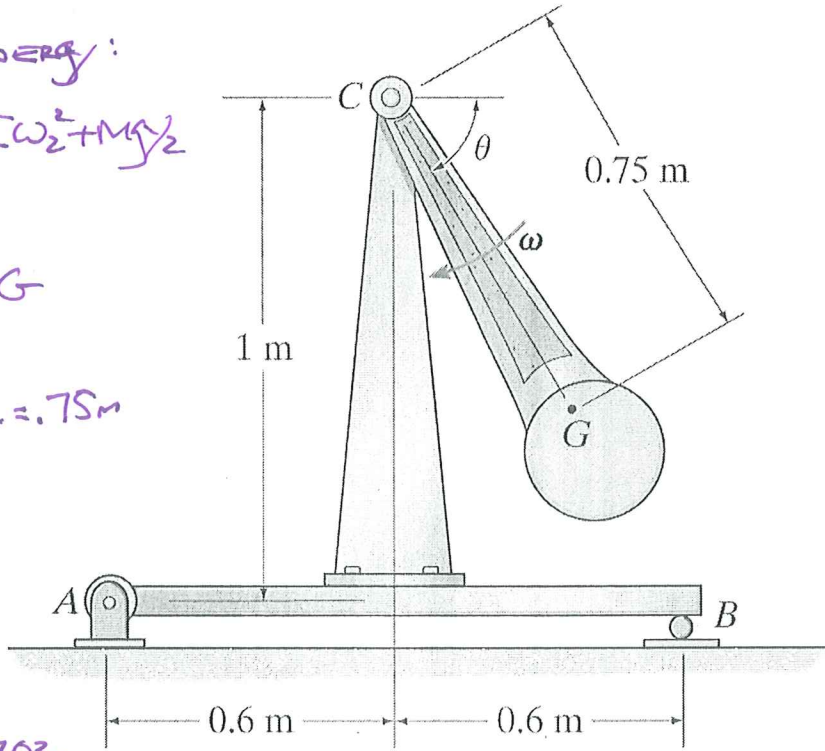
$y_1, y_2 = \text{HEIGHT OF } G$



$$y_2 = 0, y_1 = L - L \sin 65^\circ$$

$$y_1 = L(1 - \sin 65^\circ) =$$

$$0.75 \text{ m} (1 - \sin 65^\circ) = 0.0703 \text{ m}$$



7. What is the angular acceleration of the physical pendulum the instant it is released from rest at $\theta = 65^\circ$?

← NEXT PAGE

$$Mgy_1 = \frac{1}{2} I \omega_2^2$$

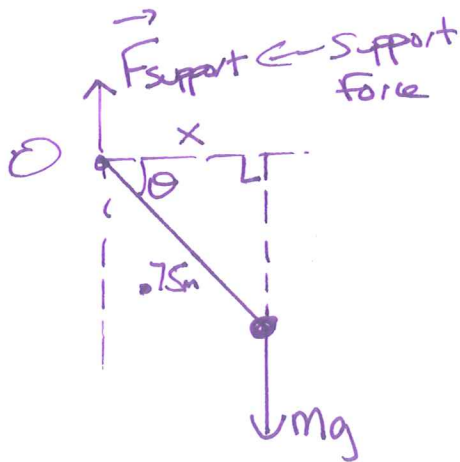
$$\Rightarrow 12 \text{ kg} (9.8 \text{ m/s}^2) (0.0703 \text{ m}) = \frac{1}{2} (5.5 \text{ kg} \cdot \text{m}^2) \omega_2^2$$

$$\Rightarrow 8.264 \text{ J} = \frac{1}{2} (5.5 \text{ kg} \cdot \text{m}^2) \omega_2^2$$

$$\Rightarrow \omega_2 = \sqrt{\frac{2(8.264 \text{ J})}{5.5 \text{ kg} \cdot \text{m}^2}} = 1.73 \text{ rad/s}^2$$

7. What is ANGULAR ACCELERATION, instant Released?

$$\sum \vec{\tau} = I \vec{\alpha}$$



Locate weight at G

Support force at $r=0$

\Rightarrow No TORQUE

$$\therefore \sum \tau = \tau_g$$

τ_g gravity

For gravity: $\tau_g = xMg$

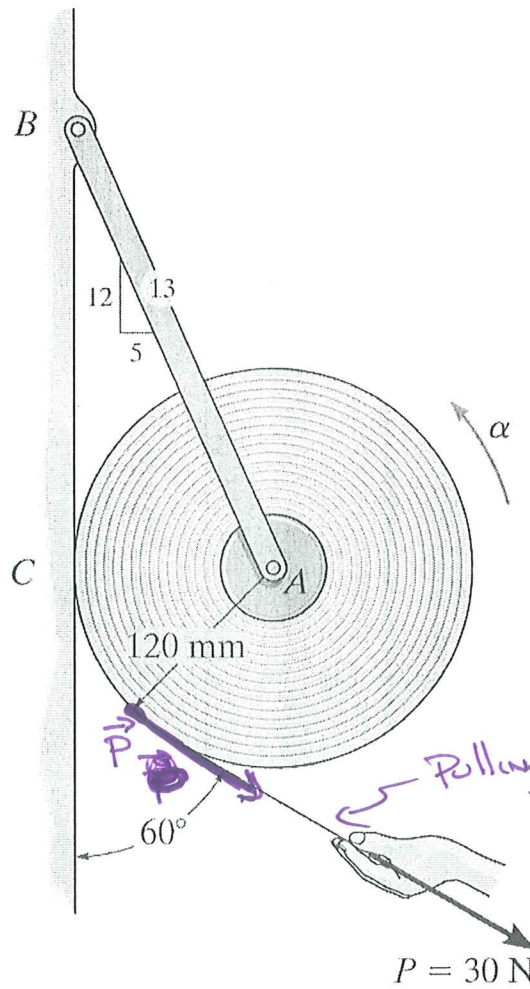
$$x = 0.75m \cos \theta = 0.75m \cos 65^\circ = 0.317m$$

$$\tau_g = I\alpha \Rightarrow \alpha = \frac{\tau_g}{I} = \frac{xMg}{I} = \frac{(0.317m)(12kg)(9.8m/s^2)}{5.5kg \cdot m^2}$$

$$\Rightarrow \alpha = 6.7772 \text{ rad/s}^2 = 6.78 \text{ rad/s}^2$$

↑
INSERT WHEN
NEEDED

8. If the roll of paper towels shown below has mass 25 kg and the coefficient of kinetic friction between the wall and paper towels is $\mu_k = 0.3$, what is the force exerted by the support rod and the angular acceleration of the roll. Treat the roll as a solid cylinder, and therefore has moment of inertia, $I = \frac{1}{2}MR^2$.



Pulling force makes tension here = 30N.
 So also have \vec{P} tangent to circle
 $P = 30\text{ N}$

Work on next page

$$F_k = \mu_k n = 0.3 \left[\frac{5}{13} F_R - 25.98 \text{ N} \right]$$

$$= \frac{1.5}{13} F_R - 7.794 \text{ N}$$

$$\therefore \frac{12}{13} F_R + \frac{1.5}{13} F_R - 7.794 \text{ N} - (25 \text{ kg})(9.8 \text{ m/s}^2) - 30 \text{ N} \cos 60^\circ = 0$$

$$\Rightarrow \frac{13.5}{13} F_R - 267.794 \text{ N} = 0$$

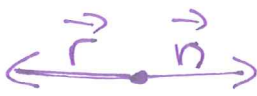
$$\Rightarrow F_R = \frac{(267.794 \text{ N})(13)}{13.5} = 257.8757 \text{ N}$$

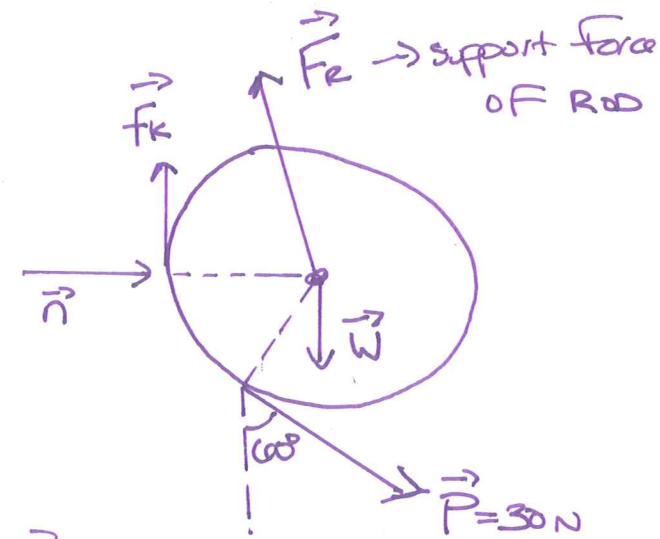
$$F_k = \frac{1.5}{13} (257.8757 \text{ N}) - 7.794 \text{ N} = 21.96 \text{ N}$$

(and $n = 73.2 \text{ N}$)

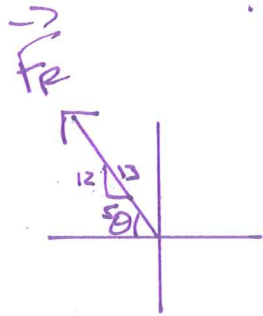
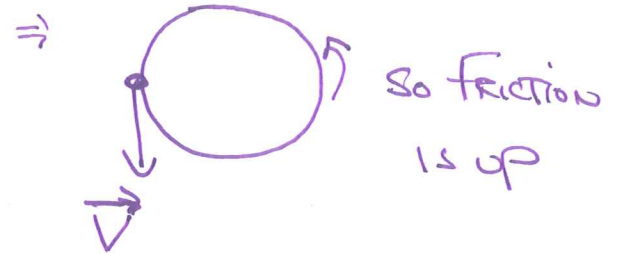
$$\sum \vec{\tau} = I \vec{\alpha} \Rightarrow \underbrace{\vec{\tau}_p}_P + \underbrace{\vec{\tau}_f}_{\text{friction}} = I \vec{\alpha}, \quad \vec{n}, \vec{\omega}, \text{ AND } \vec{F}_R$$

\vec{F}_R CAUSE NO TORQUE
 $\vec{\omega}$ at Center
 $\Rightarrow \vec{r} = 0$
 \vec{n} at 180° to \vec{r}
 $\sin 180^\circ = 0$





\Rightarrow P tries to Cause Counter-clockwise Rotation



$$\Rightarrow \theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ$$

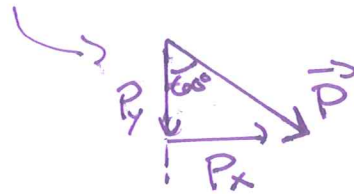
but better to use $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

$$F_{R,x} = -F_R \cos\theta = -\frac{5}{13} F_R, \quad F_{R,y} = +F_R \sin\theta = \frac{12}{13} F_R$$

Center at Rest $\Rightarrow \sum F_x = 0, \sum F_y = 0$

$$\sum F_x = 0 \Rightarrow -\frac{5}{13} F_R + n + P \sin 60^\circ = 0$$

$$\Rightarrow \frac{5}{13} F_R = n + P \sin 60^\circ$$

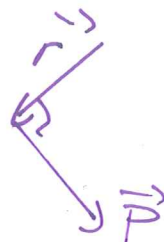


$$\frac{5}{13} F_R = n + 30 \text{ N} \sin 60^\circ = n + 25.98 \text{ N}$$

OR better yet $n = \frac{5}{13} F_R - 25.98 \text{ N}$

$$\sum F_y = 0 \Rightarrow \frac{12}{13} F_R + f_k - W - P \cos 60^\circ = 0$$

for friction  $\Rightarrow \vec{\tau}_f = r f_k \sin 90^\circ, \odot$
 $\vec{\tau}_f = r f_k, \odot$

for \vec{P}  \vec{P} tangent to circle \Rightarrow
 $\vec{\tau}_P = r P \sin 90^\circ, \odot = r P, \odot$

$$\therefore \sum \tau = r P - r f_k = r (P - f_k)$$

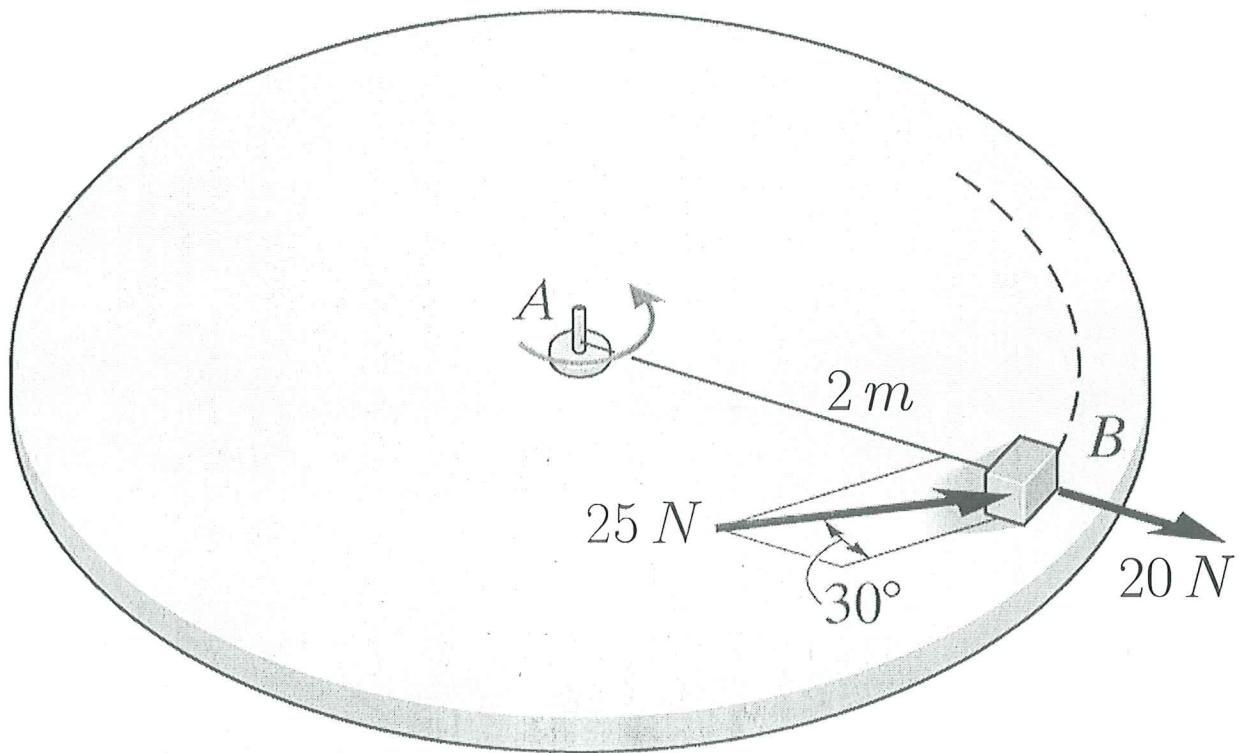
$$r = 120 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 0.12 \text{ m} \quad \Rightarrow \sum \tau = 0.12 \text{ m} (30 \text{ N} - 21.96 \text{ N})$$

$$= 0.12 \text{ m} (8.04 \text{ N}) = 0.9648 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{2} M R^2 = \frac{1}{2} (25 \text{ kg}) (.12 \text{ m})^2 = 0.18 \text{ kg}\cdot\text{m}^2$$

$$\sum \tau = I \alpha \quad \Rightarrow \quad \alpha = \frac{\sum \tau}{I} = \frac{0.9648 \text{ N}\cdot\text{m}}{0.18 \text{ kg}\cdot\text{m}^2} = 5.36 \text{ rad/s}^2$$

9. Find the torque exerted by the 25 N and 20 N forces shown. If the torques remain constant and the 3-kg mass has an initial speed of 3.4 m/s , how fast will it be going after 0.25 s ? Assume the mass continues on its 2-m circle.



For 20 N force $\tau = 0$ $\vec{r} \rightarrow$ 20 N $\phi = 0^\circ$
 $\sin 0^\circ = 0$

For 25 N force:

The first diagram shows a radial vector $r = 2\text{ m}$ pointing right, a 25 N force vector pointing up and right at 30° to the radial line, and a dashed tangent line perpendicular to the radial line. The angle between the force and the tangent line is 60° . The second diagram shows the force vector $25\text{ N} = \vec{F}$ and the radial vector \vec{r} with the 60° angle between them.

$$\Rightarrow \tau = rF \sin 60^\circ = (2\text{m})(25\text{N})\sin 60^\circ = 43.3\text{N}\cdot\text{m}$$

$$\sum \tau = 0 + 43.3\text{N}\cdot\text{m} = 43.3\text{N}\cdot\text{m}$$

Constant TORQUE $\Rightarrow \sum \tau = \frac{\Delta L}{\Delta t}$ will work

Point Particle going on A Circle $\Rightarrow L = MVR$

$$\therefore \sum \tau = \frac{MV_2 r_2 - MV_1 r_1}{\Delta t}$$

$$V_2 = ?, V_1 = 3.4\text{m/s}$$

$$r_1 = r_2 = 2\text{m}, \Delta t = 0.25\text{s}$$

$$\therefore \sum \tau = \frac{Mr(V_2 - V_1)}{\Delta t} \Rightarrow 43.3\text{N}\cdot\text{m} = \frac{3\text{kg}(2\text{m})(V_2 - 3.4\text{m/s})}{0.25\text{s}}$$

$$\Rightarrow V_2 - 3.4\text{m/s} = \frac{(43.3\text{N}\cdot\text{m})(0.25\text{s})}{3\text{kg}(2\text{m})} = 1.8\text{m/s}$$

$$\Rightarrow \boxed{V_2 = 5.2\text{m/s}}$$

NOTE you could also use $\sum \tau = I\alpha$. $I = Mr^2$ for point particle. Find ω using $\omega = \omega_0 + \alpha t$. then convert to LINEAR. $v = r\omega$ \rightarrow