Five Easy Pieces

$\Sigma F_y = ma_y$

$F - 9.8N = (1kg)(2.3m/s^2)$

$\Rightarrow F = 12.1N$

$W = \vec{F} \cdot \vec{s}$

$= (12.1N)(2m)\cos 0^\circ$

$= 24.2J$

1. A 1.0 kg mass is lifted vertically two meters (on Earth) with a constant 2.3 m/s^2 acceleration. How much work is done by the lifting force, $\vec{F}$?

(a) $W = 19.6\,J$	(b) $W = -19.6\,J$	(c) $W = 24.2\,J$	(d) $W = -24.2\,J$

2. An 80 kg man has an apparent weight of 1504 N at the bottom of a circular dip. If his speed is 6.0 m/s, what is the radius of the circle?

$\Sigma F_y = ma_y$

$n - W = \frac{mv^2}{r}$

(a) $r = 9\,m$	(b) $r = 1.26\,m$	(c) $r = 4\,m$	(d) $r = 9.8\,m$

$1504N - (80kg)(9.8m/s^2) = \frac{(80kg)(6m/s)^2}{r} \Rightarrow 720 = \frac{(80kg)(6m/s)^2}{r} \Rightarrow r = 4m$

3. A 700 kg car is traveling with a speed of 30 m/s. If 5.2 s later, its speed is 19.6 m/s, how much work, in total, was done to the car? Note the use of $kJ = kiloJoules$ to make the numbers smaller.

(a) $W = 1770\,kJ$	(b) $W = -35\,kJ$	(c) $W = 9.8\,kJ$	(d) $W = -180\,kJ$

$W_{TOTAL} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$= \frac{1}{2}(700kg)(19.6m/s)^2 - \frac{1}{2}(700kg)(30m/s)^2$

$= -180544J = -180.544kJ \approx -180kJ$

Work, Work, Work

Questions 6 through 10 refer to the following setup.

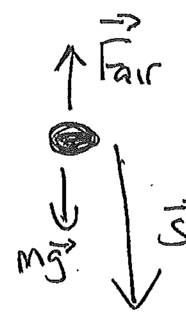
Émilie du Châtelet drops a .25 kg ball from rest, 1.6 m above a sand pit.

4. ~~6.~~ If the ball hits the sand going 3.2 m/s, how much work was done by air resistance during the ball's fall?

(a) 1.28 J	(b) $-3.92\ J$	$\boxed{\text{(c) } -2.64\ J}$	(d) $-1.28\ J$

5. ~~7.~~ Ignoring gravity, how much work must the sand do in order to stop the 3.2 m/s ball?

(a) 1.28 J	(b) $-3.92\ J$	(c) $-2.64\ J$	$\boxed{\text{(d) } -1.28\ J}$



$$W_{TOTAL} = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

$$W_{TOTAL} = W_g + W_{air} = mgs + W_{air}$$

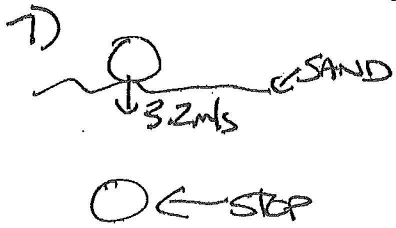
$$\Rightarrow mgs + W_{air} = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

$$S = 1.6m,\ W_{air} = ?,\ V_2 = 3.2m/s,\ V_1 = 0$$

$$\Rightarrow (.25kg)(9.8m/s^2)(1.6m) + W_{air} = \tfrac{1}{2}(.25kg)(3.2m/s)^2$$

$$\Rightarrow W_{air} = 1.28J - 3.92J = -2.64J$$

7)

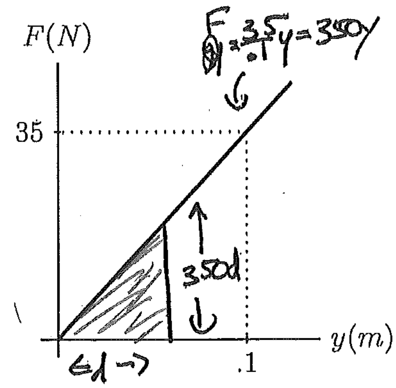


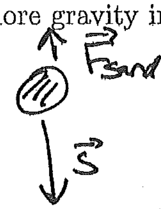
$$W_{TOTAL} = W_{SAND} = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

$$\Rightarrow W_{SAND} = 0 - 1.28J = -1.28J$$

6

8. If the force exerted by the sand increases linearly with depth below the sand, as shown on the graph below ($y = 0$ is at the top of the sand pit and down is positive), how far will the ball sink below the surface before stopping? Again, ignore gravity in this calculation.





$\Rightarrow W_{sand} = -\text{AREA}$

$\Rightarrow -1.28J = -\frac{1}{2}(d)(350d)$

$\Rightarrow -1.28J = \frac{-350}{2}d^2$

$\Rightarrow d = \sqrt{\frac{1.28(2)}{350}} = .085524$

(a) $.037\,m$	(b) $.191\,m$	$\boxed{\text{(c) } .086\,m}$	(d) $.060\,m$

7

9. If the sand provides $24\,Watt$ of average power, estimate the time it takes for the ball to stop.

$\boxed{\text{(a) } .053\,s}$	(b) $.025\,s$	(c) $9.8\,s$	(d) $3\,s$

$P_{AV} = \frac{W}{\Delta t}$

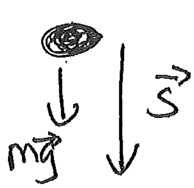
$\Rightarrow 24\,Watt = \frac{1.28J}{\Delta t}$

$\Rightarrow \Delta t = .053\,s$

8

10. If we were to include the effect of gravity in the previous calculation, the ball would:

$\boxed{\text{(a) go deeper into the sand.}}$	(b) go the same distance into the sand.
(c) go less distance into the sand.	(d) bounce off the sand.

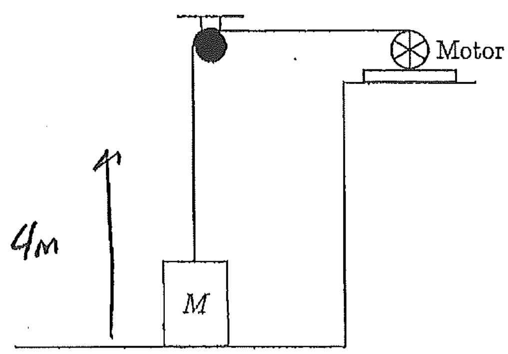


$W_g = m\vec{g}\cdot\vec{s} = mgs\cos 0° = mgs$

$\Rightarrow$ GRAVITY DOING POSITIVE WORK SO BALL would go farther

Lifting

One day finds your instructor needing to lift a 10.0 kg box. As usual, he has completely over-complicated the procedure by using a massless pulley and a motor as shown below. Your instructor observes that when the motor has lifted the box 4.0 m above the floor, it has a speed of 5.0 m/s.



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18. How much gravitational potential energy does the box have? (Assume the on the floor, the box had zero potential energy.)

(a) 98 J	(b) 4 J	(c) 392 J	(d) 400 J

$U_g = Mgy$
$= (10kg)(9.8m/s^2)(4m)$
$= 392J$

10

19. How much kinetic energy does the box have?

(a) 250 J	(b) 392 J	(c) 517 J	(d) 125 J

$K = \frac{1}{2}mv^2 = \frac{1}{2}(10kg)(5m/s)^2$
$= 125J$

$$\frac{1}{2}m\cancel{v_1^2}^{\,0} + \cancel{mgy_1}^{\,0} + W_{motor} = \underbrace{\frac{1}{2}mv^2}_{125J} + \underbrace{mgy_2}_{392J}$$

$$\Rightarrow W_{motor} = 125J + 392J = 517J$$

11 ~~20~~. Assuming the box started from rest, how much work was done by the motor lifting the box to $4\,m$?

(a) $267\,J$	(b) $392\,J$	(c) $517\,J$	(d) $125\,J$

12 ~~21~~. If at the instant the box is $4\,m$ above the ground the motor is supplying $640\,Watt$ of power, what is the box's acceleration? **HINT**: Use the equation $P = \vec{F} \cdot \vec{v}$ where $\vec{F}$ is the force the motor exerts on the rope and $\vec{v}$ is the velocity of the rope entering the motor.

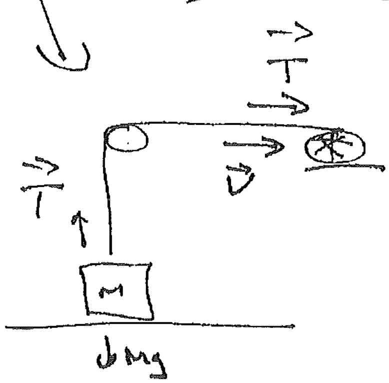
(a) $9.8\,m/s^2$	(b) $0\,m/s^2$	(c) $128\,m/s^2$	(d) $3\,m/s^2$

13 ~~22~~. Assuming constant acceleration, which of the following is a true statement about the average power, P_{av} supplied by the motor while the box was rising $4\,m$?

(a) $P_{av} = 640\,Watt$	(b) $P_{av} > 640\,Watt$
(c) $P_{av} < 640\,Watt$	(d) There is not enough information to determine.

$P = TV$ at All times.
Constant Acceleration
$\Rightarrow$ V INCREASING
$\Rightarrow$ P WAS INCREASING,

So P started at 0 AND INCREASED to 640Watt $\Rightarrow$ THE AVERAGE WAS less than 640Watt



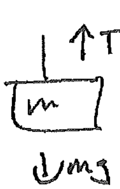
PERFECT PULLEY $\Rightarrow$ FORCE EXERTED by Motor = tension

ONE ROPE $\Rightarrow$ $\vec{v}$ of ROPE = $\vec{v}$ of MASS

$\Rightarrow V = 5m/s$

$\vec{v} \rightarrow \ \vec{T} \rightarrow \qquad P = \vec{T}\cdot\vec{v} = TV\cos 0^\circ = TV$

$$\Rightarrow T = \frac{P}{V} = \frac{640\,Watt}{5\,m/s} = 128N$$

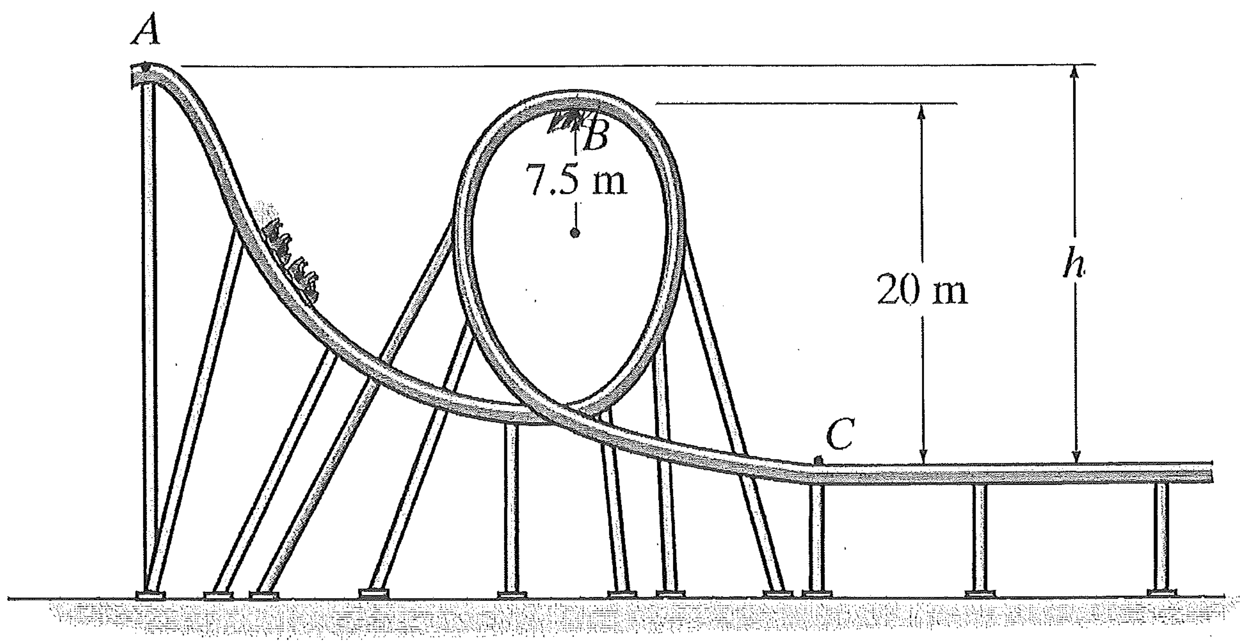


$$\sum F_y = Ma_y \Rightarrow T - mg = ma_y \Rightarrow 128N - (10kg)(9.8m/s^2) = 10kg\,a_y$$

$$\Rightarrow a_y = \frac{30N}{10kg} = 3m/s^2$$

11.

(11) A roller coaster starts from rest at point A where the height $h = 25\,m$. It slides along the track without friction. What is the normal force acting on a $75\,kg$ rider of the roller coaster at the top of the loop-to-loop (point B)? As shown, the radius of the loop-to-loop's circle is $7.5\,m$.



(a) $0\,N$	(b) $1715\,N$	(c) $245\,N$	(d) The roller coaster doesn't make it to B

AT TOP OF loop-to-loop, BOTH weight AND NORMAL Point DOWN.

Centripetal Acceleration Also points DOWN.

FORCES ON 75kg RIDER:

$\vec{n}\downarrow\ \downarrow\vec{w}$ $\quad n = ?,\ w = (75kg)(9.8m/s^2) = 735N.\quad a_y = -a_c$

$\downarrow\vec{a}_c$ $\quad \sum F_y = ma_y \Rightarrow -n - 735N = 75kg(-a_c)$

$-n - 735N = -75kg\, a_c$

$\Rightarrow n + 735N = 75kg\, a_c. \Rightarrow n = (75kg)a_c - 735N$

$a_c = \frac{v^2}{r} = \frac{v^2}{7.5m}$ ← If we find V at B, we can find a_c and then n.

From A to B, gravity only force doing work

$\Rightarrow \frac{1}{2}\cancel{m}v_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$

At A: $v_i = 0$, $y_i = h = 25m$,

At B: $v_f = ?$, $y_f = 20m$

$\cancel{m}g(25m) = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}g(20m)$

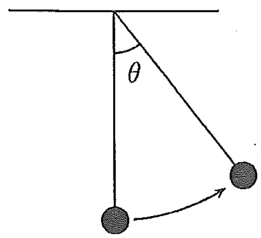
$\Rightarrow g(25m - 20m) = \frac{1}{2}v_f^2$

$\Rightarrow v_f^2 = 2g(25m - 20m) = 2(9.8m/s^2)(5m) = 98\,m^2/s^2$ $\left\{ v_f = \sqrt{98} = 9.8994 \right.$

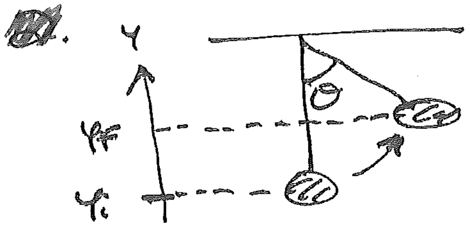
$a_c = \frac{v^2}{r} = \frac{98\,m^2/s^2}{7.5m} = 13.0667\,m/s^2$

$\Rightarrow n = (75kg)(13.0667\,m/s^2) - 735N = 980N - 735N = 245N$

15
(21) A 1.65-m long pendulum is started from the vertical position by giving it a speed of $0.900\,m/s$. To what maximum angle θ (from the vertical) will the pendulum go before turning around?



(a) 88.6°	(b) 77.2°
(c) 34.3°	$\boxed{\text{(d) 12.9°}}$



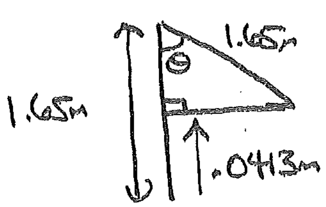
Gravity only force doing work ⇒

$$\frac{1}{2}mv_i^2 + \cancel{mgy_i} = \frac{1}{2}m\cancel{v_F^2} + mgy_F$$

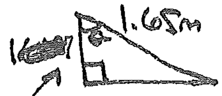
$V_i = .9\,m/s,\ V_F = 0$

Let $y_i = 0,\ y_F = ?$

$$\frac{1}{2}\cancel{m}(.9\,m/s)^2 = \cancel{m}(9.8\,m/s^2)y_F \Rightarrow y_F = \frac{(.9\,m/s)^2}{2(9.8\,m/s^2)} = 0.0413\,m$$



⇒

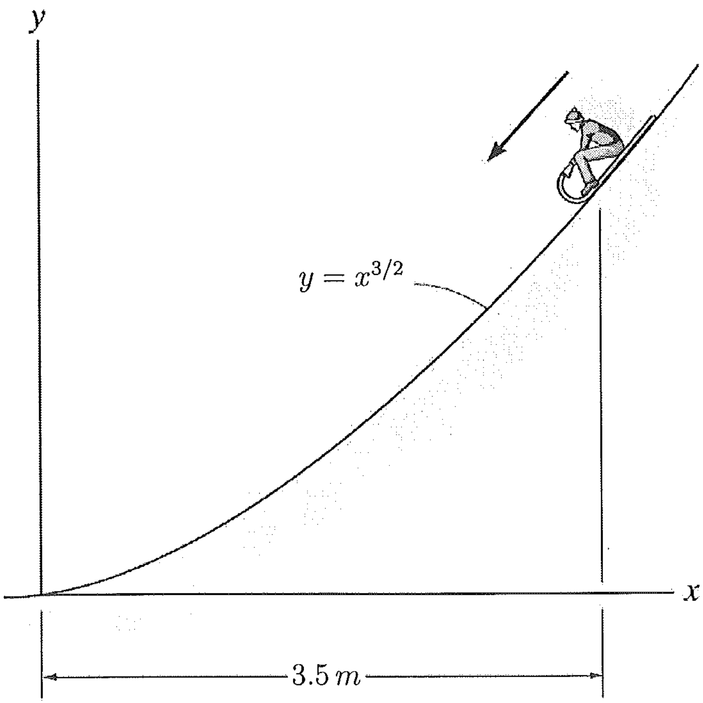


$1.65m - .0413m$
$= 1.6087m$

$$\cos\theta = \frac{1.6087m}{1.65m} = 0.97495$$

$$\Rightarrow \theta = \cos^{-1}(0.97495) = 12.85°$$
$= 12.9°$

16. A boy rides a sled down an icy (and therefore frictionless) hill whose height above the ground is given by the equation $y = x^{3/2}$, where y is in meters when x is in meters. If he starts from rest at $x = 3.5\,m$, how fast will he be going at the bottom?



(a) $0\,m/s$	(b) $8.28\,m/s$
(c) $5.42\,m/s$	(d) $11.3\,m/s$

Gravity only Force Doing Work $\Rightarrow$

$\frac{1}{2}m\cancel{v_1^2} + mgy_1 = \frac{1}{2}mv_2^2 + \cancel{mgy_2}$

$V_1 = 0, \; V_2 = ?$

Gravity only Cares About Height $\Rightarrow$ $y_1 = (3.5m)^{3/2} = 6.5479m$

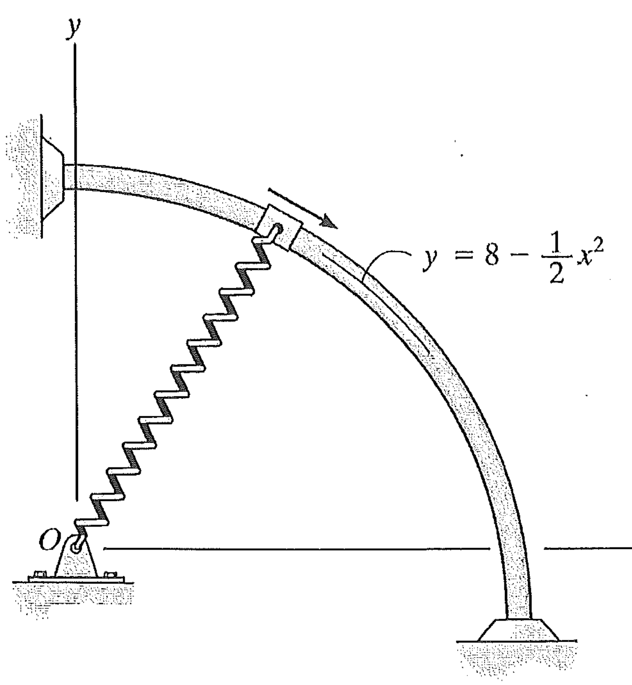
$y_2 = 0$

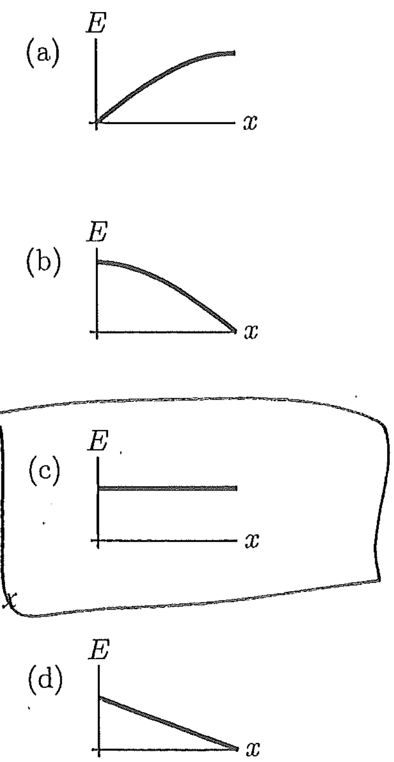
$\Rightarrow \cancel{m}gy_1 = \frac{1}{2}\cancel{m}V_2^2 \Rightarrow V_2 = \sqrt{2gy_1} = \sqrt{2(9.8\,m/s^2)(6.5479\,m)}$

$= 11.3\,m/s$

17.

A 6 kg collar is allowed to slide over a frictionless pole whose height above the ground obeys the parabolic equation $y = 8 - (1/2)\,x^2$, where y is in meters when x is in meters. Attached to the collar is a $k = 30\,N/m$ spring. The spring, unstretched length $1\,m$, is connected such that as the collar moves, the spring is always oriented along the line connecting the point O and the collar. If the collar is started from rest at $x = 0$, which of the following graphs correctly displays the collar's total energy, E, versus position, x as it slides down the pole? BONUS: FIND Speed





ONLY gravity AND Spring DO WORK ⇒ only Conservative forces ⇒ Total Energy Conserved

Bonus: $\frac{1}{2}mV_1^2 + mgy_1 + \frac{1}{2}KS_1^2 = \frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}KS_2^2$

$V_1 = 0, \quad V_2 = ?$

Starts at $x=0 \Rightarrow y_1 = 8 - 0 = 8m$

$S_1 = l_1 - l_0$ at y_1 spring is straight up $\Rightarrow$ $l = y_1 = 8m$

$l_0 = 1m \Rightarrow S_1 = 7m$

at V_2, $y=0$, $\quad 0 = 8 - \frac{1}{2}x^2 \Rightarrow x^2 = 16 \Rightarrow x = 4m$

spring is horizontal there $\Rightarrow$ $l = x = 4m$

$\Rightarrow S_2 = 4m - 1m = 3m$

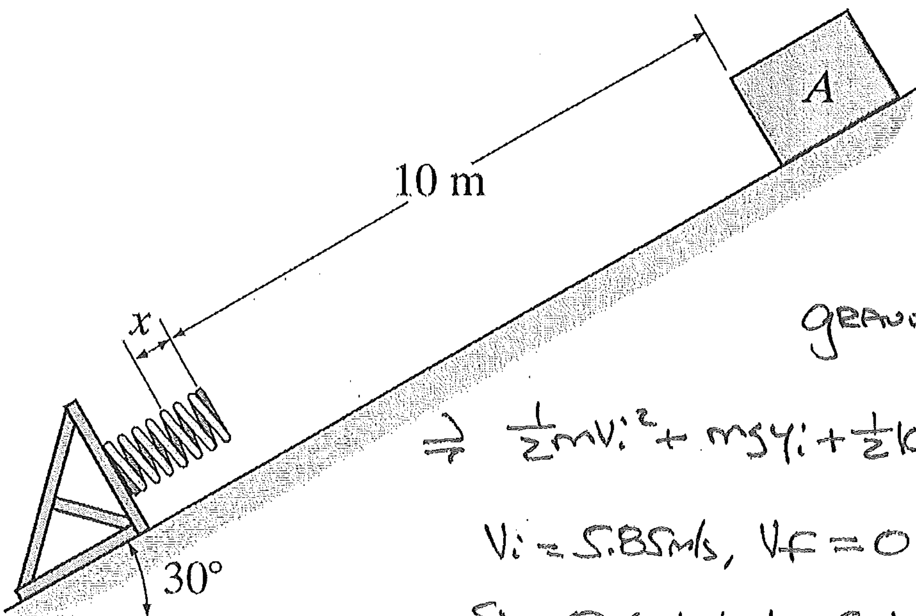
$\therefore\ 0 + (6kg)(9.8m/s^2)(8m) + \frac{1}{2}(30N/m)(7m)^2 = \frac{1}{2}(6kg)V_2^2 + 0 + \frac{1}{2}(30N/m)(3m)^2$

$\Rightarrow 470.4J + 735J = \frac{1}{2}(6kg)V_2^2 + 135J$

$\Rightarrow \frac{1}{2}(6kg)V_2^2 = 470.4J + 735J - 135J = 1070.4J$

$\Rightarrow V_2 = \sqrt{\frac{2(1070.4J)}{6kg}} = 18.88915 m/s = 18.9 m/s$

18. A $12.0\,kg$ mass slides $10.0\,m$ down a $30°$ incline starting with a speed of $5.85\,m/s$ before hitting a $424\,N/m$ spring. What *additional* distance, x, does the mass travel before stopping? Assume the incline is frictionless.



gravity & Spring Do work

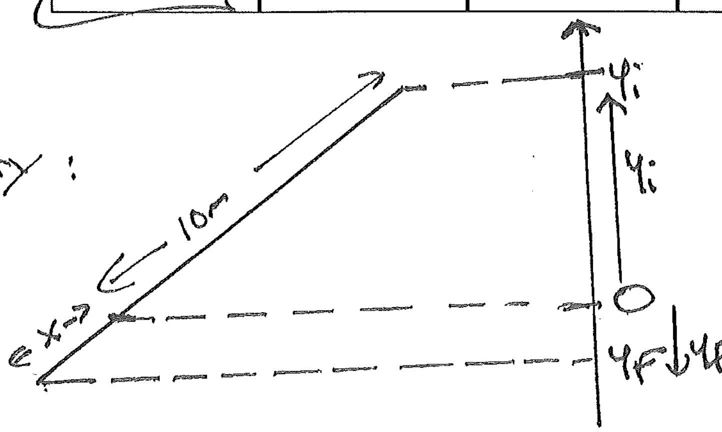
$$\Rightarrow \tfrac{1}{2}mv_i^2 + mgy_i + \tfrac{1}{2}ks_i^2 = \tfrac{1}{2}mv_f^2 + mgy_f + \tfrac{1}{2}ks_f^2$$

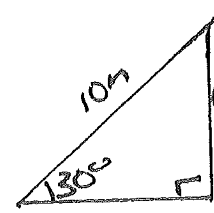
$V_i = 5.85\,m/s$, $V_f = 0$

$S_i = 0$ (Not touching Spring at A), $S_f = x = ?$

(a) $2.08\,m$	(b) $1.49\,m$	(c) $10.0\,m$	(d) $1.6\,m$

For gravity :





$y_i = 10m \sin 30° = 5m$

y_F is below zero, so must be negative

$y_F = -x\sin 30° = -x(0.5)$

$$\frac{1}{2}mv_i^2 + mgy_i + \cancel{\frac{1}{2}kS_i^2}^{\,0} = \cancel{\frac{1}{2}mv_f^2}^{\,0} + mgy_f + \frac{1}{2}kS_f^2$$

$$\frac{1}{2}(12\,kg)(5.85\,m/s)^2 + (12\,kg)(9.8\,m/s^2)(5\,m) = (12\,kg)(9.8\,m/s^2)[-(0.5x)] + \frac{1}{2}(424\,N/m)x^2$$

$$\Rightarrow 205.335\,J + 588\,J = -(58.8\,N)\,x + (212\,N/m)x^2$$

$$793.335\,J = -(58.8\,N)x + (212\,N/m)x^2$$

$$\Rightarrow (212\,N/m)x^2 - (58.8\,N)\,x - 793.335\,J = 0 \longleftarrow \text{QUADRATIC}$$

$$x = \frac{+58.8 \pm \sqrt{(58.8)^2 - 4(212)(-793.335)}}{2(212)} = \frac{+58.8 \pm \sqrt{676205.52}}{424}$$

$$\Rightarrow x = \frac{881.117}{424} = 2.0781\,m = 2.08\,m$$

or $\cancel{x = -1.8\,m}$

↑

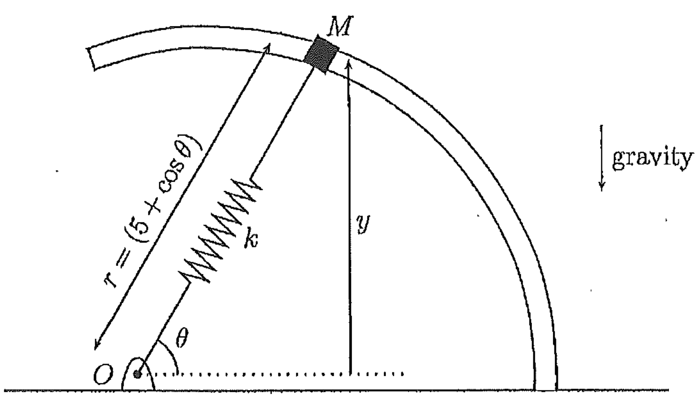
Already took care of negative sign when we set $y_f = -0.5x$

A.

23. High Art

One day whilst on a walking tour of Santa Fe, you come across a most curious piece of kinetic art. It consists of an $18\,kg$ steel collar that vertically slides over a frictionless, fancy-shaped track while attached to a $26\,N/m$ spring. As shown below, the spring, unstretched length $1.1\,m$, is connected so that it is free to swing around with the mass and is always oriented along the line connecting the collar and the point labeled O.

The *artiste* who designed the sculpture proudly tells you that she has carefully designed the track so that it has the famous shape called a limaçon. She's even able to give you the exact equation for the track's limaçon, $r = (5 + \cos\theta)$, where r is the distance (in meters) from O to the collar and θ is the angle shown below.



(a) The *artiste* informs you that her original vision had the $18\,kg$ collar starting from rest at $\theta = 90°$ and gracefully sliding down to $\theta = 0°$. She was "bummed" (to use her phrase) when the collar did no such thing. It had to be started with some minimum speed. Using methods discussed in the bonus, you find that the collar's potential energy is greatest at $\theta = 73.4°$. What minimum speed must be given to the collar at $\theta = 90°$ for it to just reach $\theta = 73.4°$? **HINT:** The mass has potential energy due to gravity and the spring. The picture indicates how to find the height, y, of the mass for any angle θ. The stretching distance of the spring is given by $r - l_0$ where l_0 is the spring's unstretched length.

GRAVITY AND Spring DO work $\Rightarrow$ $\frac{1}{2}mV_1^2 + mgy_1 + \frac{1}{2}KS_1^2 = \frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}KS_2^2$

$V_1 = ?$, $V_2 = 0$, $y = r\sin\theta = (5 + \cos\theta)\sin\theta \Rightarrow y_1 = y(\theta = 90°) = 5\text{m}$

$y_2 = \underbrace{(5 + \cos 73.4°)}_{r_2}\sin 73.4° = (5.286\text{m})\sin 73.4° = 5.065\text{m}$

$S_1 = r_1 - l_0 = 5\text{m} - 1.1\text{m} = 3.9\text{m}$

$S_2 = r_2 - l_0 = 5.286\text{m} - 1.1\text{m} = 4.186\text{m}$

$\therefore \frac{1}{2}(18\text{kg})V_1^2 + (18\text{kg})(9.8\text{m/s}^2)(5\text{m}) + \frac{1}{2}(26\text{N/m})(3.9\text{m})^2 = (18\text{kg})(9.8\text{m/s}^2)(5.065\text{m}) + \frac{1}{2}(26\text{N/m})(4.186\text{m})^2$

$\Rightarrow \frac{1}{2}(18\text{kg})V_1^2 + 882\text{J} + 197.73\text{J} = 893.47\text{J} + 227.79\text{J}$

$\Rightarrow V_1 = \sqrt{\frac{41.53\text{J}(2)}{18\text{kg}}} = 2.148\text{m/s} = 2.15\text{m/s}$

(b) During one particulary memorable run of the sculpture, the collar went from $\theta = 73.4°$ (where it was momentarily at rest) down to $\theta = 0°$ whilst a big gust of wind was blowing. (It was so impressive that your monocle nearly popped out.) If the collar reached $\theta = 0°$ with a speed of $6.5\,m/s$, how much work was done by that gust of wind?

$$\frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}kS_2^2 + W_{air} = \frac{1}{2}mV_3^2 + mgy_3 + \frac{1}{2}kS_3^2$$

$V_2 = 0$, $y_2 = 5.065\,m$, $S_2 = 4.186\,m$, $V_3 = 6.5\,m/s$, $y_3 = 0$, $S_3 = (5+\cos 0°) - 1.1 = 6m - 1.1m = 4.9m$

$$893.47\,J + 227.79\,J + W_{air} = \frac{1}{2}(18\,kg)(6.5\,m/s)^2 + \frac{1}{2}(26\,N/m)(4.9\,m)^2$$

$$\Rightarrow 1121.26\,J + W_{air} = 380.25\,J + 312.13\,J$$

$$\Rightarrow \boxed{W_{air} = -428.88\,J = -429\,J}$$

(c) **BONUS:** Show that the potential energy of the collar has its maximum value at the point where $\theta = 73.4°$. **HINT:** Find the potential energy as a function of theta. Use your new/old-found calculus skills to find the maximum.

<u>Bonus</u>: $U = mgy + \frac{1}{2}ks^2$

$$U = (18)(9.8)\, r\sin\theta + \tfrac{1}{2}(26)\left[r - l_0\right]^2$$

$$= 176.4\,(5+\cos\theta)\sin\theta + 13\left[5+\cos\theta - 1.1\right]^2$$

$$= 176.4\,(5+\cos\theta)\sin\theta + 13\left[3.9+\cos\theta\right]^2$$

MAX $\Rightarrow \frac{dU}{d\theta} = 0$, $\frac{dU}{d\theta} = 176.4\,(5+\cos\theta)\cos\theta + 176.4\,(-\sin\theta)\sin\theta$

$$+ 13\,(2)\left[3.9+\cos\theta\right]^1 \cdot (-\sin\theta)$$

$$= 176.4(5+\cos\theta)\cos\theta - 176.4\sin^2\theta - 26\left[3.9+\cos\theta\right]\sin\theta$$

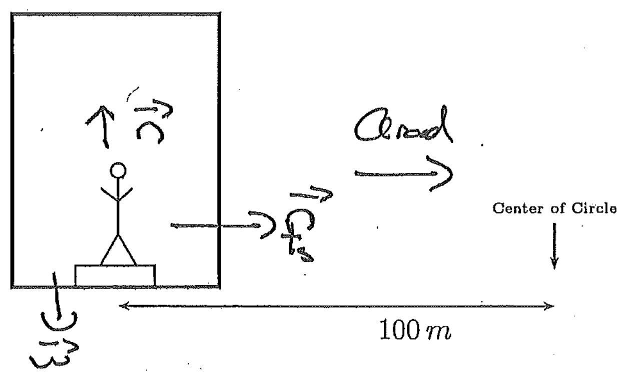
So check, at $\theta = 73.4^\circ$

$$\frac{dU}{d\theta} = 176.4(5+\cos 73.4^\circ)\cos 73.4^\circ - 176.4\sin^2 73.4^\circ - 26\left[3.9+\cos 73.4^\circ\right]\sin 73.4^\circ$$

$$= 266.37 - 162 - 104.3 = .077 \approx 0$$

~~21~~ 20. The Great Glass Elevator

One day finds little Charlie Bucket (mass 48 kg) and Willy Wonka riding around (on Earth) in their fabulous great glass elevator. If you've never read the book, the great glass elevator is an elevator that can move in any direction you might wish. Sometime during their trip, little Charlie Bucket and Willy Wonka turn a corner in the great glass elevator by zooming around a 100 m radius circle with a speed of 22.2 m/s. For reasons that only make sense to Willy Wonka (and your instructor), Charlie is riding in the elevator standing on a scale.



(a) If the coefficient of static friction between Charlie Bucket and his scale is .39, will he be able to remain not-sliding as he travels around this circle? Assume, as shown above, that the center of the circle is directly to the right of Charlie Bucket. (You must do a calculation of some sort to get full credit on this problem which is why there is an extra page provided.) *(10pts)*

Forces on Charlie: $\vec{n}$ up, $\vec{w}$ down, $\vec{f_s}$ right

AND NO OTHERS!

Circle's Center to RIGHT $\Rightarrow a_{rad} = a_x$ to right

$$\Rightarrow \sum F_x = ma_x \Rightarrow f_s = ma_{rad} = \frac{mv^2}{r}$$

$$\Rightarrow f_s = \frac{(48kg)(22.2m/s)^2}{100m} = 236.5632N$$ $\Rightarrow$ Need 237N of static friction to go AROUND Circle.

$f_{s,max} = \mu_s n$ $\quad \sum F_y = ma_y.$ $\quad a_y = 0$

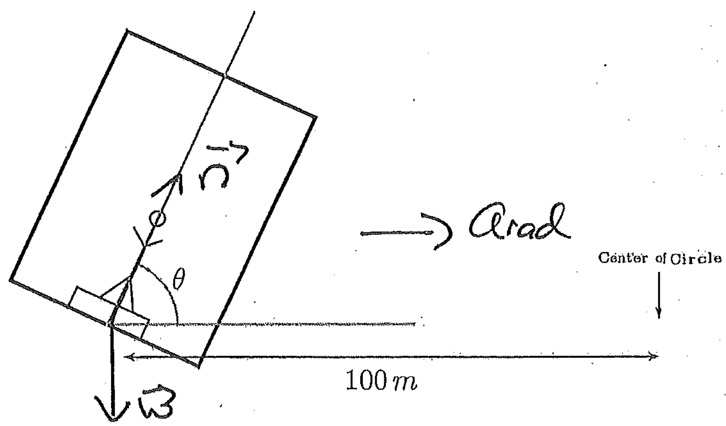
$$\Rightarrow n - W = 0 \Rightarrow n = W = (48kg)(9.8m/s^2)$$
$$= 470.4N$$

$$\Rightarrow f_{s,max} = .39(470.4N) = 183.456N$$

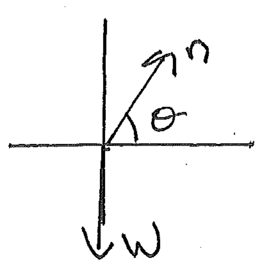
Since static friction's MAX VALUE is only 183N AND CHARLIE Needs 237N to go AROUND Circle He will SLIDE!

$\hookrightarrow$ OUCH

(b) By fiddling with some buttons, Willy Wonka discovers that he can tilt the great glass elevator to any angle that he wishes. To what angle θ should Willy Wonka tilt the elevator so that no friction is necessary for Charlie Bucket to go around a 100 m radius circle (whose center is still directly to his right) with a speed of 22.2 m/s? What would the scale read in this case? (There's an extra page for this one too.) *(10pts)*



Now $\vec{n}$ at Angle θ, $\vec{W}$ DOWN, No Friction. $a_{rad} = a_x$ still



$\sum F_x = ma_x \Rightarrow n\cos\theta = \frac{mv^2}{r}$

$\sum F_y = ma_y \Rightarrow n\sin\theta - W = 0 \Rightarrow n\sin\theta = mg$

$n\sin\theta = mg, \quad n\cos\theta = \frac{mv^2}{r}$

$\Rightarrow \frac{\cancel{n}\sin\theta}{\cancel{n}\cos\theta} = \frac{\cancel{m}g}{\cancel{m}v^2/r} \Rightarrow \tan\theta = \frac{gr}{v^2} \Rightarrow \theta = \tan^{-1}\left(\frac{gr}{v^2}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{9.8\,m/s^2 \cdot 100\,m}{(22.2\,m/s)^2}\right) = \tan^{-1}(1.988) \Rightarrow \boxed{\theta = 63.3^\circ}$

$n\sin\theta = mg \Rightarrow n = \frac{mg}{\sin\theta} = \frac{(48\,kg)(9.8\,m/s^2)}{\sin 63.3^\circ} \Rightarrow \boxed{n = 526.5\,N = 527\,N}$

$\Rightarrow m = \frac{526.5}{9.8} = 53.7\,kg$