(1.) What is the magnitude and direction of the vector sum for the two force vectors shown.

\[ \Sigma F_x = F_{1,x} + F_{2,x}, \quad \Sigma F_y = F_{1,y} + F_{2,y} \]

\[ F_{1,x} = +F_1 \cos 45^\circ \]
\[ F_{1,y} = -F_1 \sin 45^\circ \]

\[ F_{2,x} = -F_2 \cos 60^\circ \]
\[ F_{2,y} = -F_2 \sin 60^\circ \]

\[ \Sigma F_x = 2kN \cos 45^\circ - 6kN \cos 60^\circ = -1.586kN \]
\[ \Sigma F_y = -2kN \sin 45^\circ - 6kN \sin 60^\circ = 6.61kN \]

\[ \Sigma F = \sqrt{(1.586kN)^2 + (6.61kN)^2} = 6.8kN \]

\[ \theta = \tan^{-1} \left( \frac{6.61kN}{1.586kN} \right) = 76.5^\circ \]
(2.) A projectile is launched at $37^\circ$ above the horizontal from the top of the cliff with a speed of $25 \text{ m/s}$. If its range is $73.6 \text{ m}$, how high above the ground is the cliff?

\begin{align*}
\begin{array}{|c|}
\hline
\text{(a)} & 9.6 \text{ m} \\
\text{(b)} & 11.8 \text{ m} \\
\text{(c)} & 15.1 \text{ m} \\
\text{(d)} & 28.8 \text{ m} \\
\text{(e)} & 31.5 \text{ m} \\
\hline
\end{array}
\end{align*}

**Known:** $x_0 = 0$, $x = 73.6 \text{ m}$, $y = 0$

\begin{align*}
V_{0x} &= V_0 \cos \alpha = 25 \text{ m/s} \cos 37^\circ \\
V_{0y} &= V_0 \sin \alpha = 25 \text{ m/s} \sin 37^\circ
\end{align*}

**Unknown:** $y_0 = h = ?$, $t$, $V_y$

So use $x = x_0 + V_{0x} t$ to find $t$

Then use $y = y_0 + V_{0y} t - \frac{1}{2}gt^2$ to find $h$

\begin{align*}
x &= x_0 + V_{0x} t \\&= 73.6 \text{ m} = 25 \text{ m/s} \cos 37^\circ t \\& \Rightarrow t = \frac{73.6 \text{ m}}{25 \text{ m/s} \cos 37^\circ} = 2.97 \text{ s}
\end{align*}

\begin{align*}
y &= y_0 + V_{0y} t - \frac{1}{2}gt^2 \\
&= 0 + (25 \text{ m/s}) \sin 37^\circ (2.97 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(2.97 \text{ s})^2 \\
&= 3.6865 \text{ m}
\end{align*}

\begin{align*}
\Rightarrow 0 &= h + (25 \text{ m/s}) \sin 37^\circ \left(\frac{3.6865 \text{ m}}{2.97 \text{ s}}\right) - \frac{1}{2}(9.8 \text{ m/s}^2)(2.97 \text{ s})^2 \\
\Rightarrow 0 &= h + 38.46 \text{ m} - 60.57 \text{ m} \\
\Rightarrow 0 &= h - 12.1 \text{ m}
\end{align*}

\begin{align*}
\Rightarrow h &= 12.1 \text{ m}
\end{align*}
(3.) By pulling on the 25-kg crate as shown, the man is able to drag it horizontally across the floor. Which of the following is a true statement about the crate?

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>The normal force acting on the crate is equal to 245 N.</td>
</tr>
<tr>
<td>(b)</td>
<td>The normal force acting on the crate is smaller than 245 N.</td>
</tr>
<tr>
<td>(c)</td>
<td>The normal force acting on the crate is greater than 245 N.</td>
</tr>
<tr>
<td>(d)</td>
<td>There is not enough information to make a statement about the magnitude of the normal force as compared to 245 N.</td>
</tr>
</tbody>
</table>

**Forces:** \( \vec{N} \), \( \vec{W} \), \( \vec{T} \) at 30°

Maybe friction, \( \vec{F}_k \) to left

\[
\begin{align*}
\sum F_y &= 0 \\
N + Ty + Wy + fky &= 0 \\
N + T \sin 30° - W &= 0 \\
N &= \omega - \frac{1}{2} T
\end{align*}
\]

\( \omega = 25 \text{kg} \times 9.8 \text{m/s}^2 = 245 \text{N} \)

So normal must be **smaller** than 245 N.
(4.) A 65 kg man riding in an elevator has an apparent weight of 559 N, what is the elevator’s acceleration?

\[ \text{Forces: } n \text{ up, } n = 559 \text{N} \]

\[ \vec{W} \text{ down, } \omega = (65 \text{ kg}) \times 9.8 \text{ m/s}^2 = 637 \text{N} \]

\[ \sum F_y = m \omega \rightarrow n - \omega = m \omega \]

\[ \Rightarrow 559 \text{N} - 637 \text{N} = (65 \text{ kg}) \omega \]

\[ \Rightarrow -78 \text{N} = (65 \text{ kg}) \omega \]

\[ \Rightarrow \omega = \frac{-78 \text{N}}{65 \text{ kg}} = -1.2 \text{ m/s}^2 \]
(5.) A boy rides a sled down an icy (and therefore frictionless) hill whose height above the ground is given by the equation $y = x^{5/2}$, where $y$ is in meters when $x$ is in meters. If he starts from rest at $x = 1.5\, m$, how fast will he be going at the bottom?

Gravity only force doing work $\Rightarrow$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

$V_i = 0, \quad y_1 = (1.5)^{5/2} = 2.756\, m, \quad V_2 = ?, \quad y_2 = 0$

$$\Rightarrow \frac{1}{2}v_2^2 = 9y_1 \Rightarrow v_2 = \sqrt{2(9.8\, m/s^2)(2.756\, m)}$$

$$= \sqrt{54.6\, m^2/s^2} = 7.35\, m/s$$
(6.) A 12.5-kg mass is sliding across a frictionless floor until it stopped by a 750 N/m spring (which is initially uncompressed). If the maximum compression of the spring is 0.15 m, how fast was the mass originally going?

Spring only force doing work \( \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}kS_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kS_2^2 \)

\[ V_1 = ?, \quad S_1 = 0, \quad V_2 = 0, \quad S_2 = 0.15 \text{ m} \]

\[ \Rightarrow \frac{1}{2}(12.5\text{ kg})(v_1^2) = \frac{1}{2} (750\text{ N/m})(0.15\text{ m})^2 \]

\[ \Rightarrow v_1 = \sqrt{\frac{(750\text{ N/m})(0.15\text{ m})^2}{12.5\text{ kg}}} = \sqrt{1.35\text{ m}^2/s^2} = 1.16\text{ m/s} \]
(7.) A 2000-kg tanker car going 5.00 m/s to the right has a completely inelastic collision with a 1000-kg boxcar going to the left. If the combination is going 1.00 m/s to the left after the collision, how fast was the boxcar going before?

\[ M_A V_{A1,x} + M_B V_{B1,x} = (M_A + M_B) V_{2,x} \]

\[ V_{A1,x} = 5 \text{ m/s}, \quad V_{B1,x} = ?, \quad V_{2,x} = -1 \text{ m/s} \]

\[ \text{Conservation of Momentum:} \]

\[ (2000 \text{ kg})(5 \text{ m/s}) + (1000 \text{ kg}) V_{B1,x} = (3000 \text{ kg})(-1 \text{ m/s}) \]

\[ \implies 10000 \text{ kg m/s} + 1000 \text{ kg} V_{B1,x} = -3000 \text{ kg m/s} \]

\[ \implies 1000 \text{ kg} V_{B1,x} = -3000 \text{ kg m/s} - 10000 \text{ kg m/s} = -13000 \text{ kg m/s} \]

\[ \implies V_{B1,x} = \frac{-13000 \text{ kg m/s}}{1000 \text{ kg}} = -13 \text{ m/s} = 13 \text{ m/s to left} \]
(8.) A point 0.65 m from the center of a spinning wheel has a linear speed of 1.3 m/s. What is the angular speed of the wheel in RPM?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.209 RPM</td>
</tr>
<tr>
<td>(b)</td>
<td>0.5 RPM</td>
</tr>
<tr>
<td>(c)</td>
<td>2 RPM</td>
</tr>
<tr>
<td>(d)</td>
<td>4.77 RPM</td>
</tr>
<tr>
<td>(e)</td>
<td>19.1 RPM</td>
</tr>
</tbody>
</table>

\[ \sqrt{V^2} = w \cdot r \Rightarrow w = \frac{V}{r} = \frac{1.3 \text{ m/s}}{0.65 \text{ m}} = 2 \text{s} \]

\[ \frac{1}{5} \Rightarrow w = \frac{2 \text{ rad}}{5} \cdot \frac{\text{rev}}{2\pi \text{ rad}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 19.1 \text{ RPM} \]
(9.) A uniform bar is leaning at rest against a wall as shown. If the normal force acting on the bar at point A is 75 N, what is the weight of the bar? Assume there is no friction between the bar and the wall at point A.

\[ \text{Uniform Bar} \Rightarrow \text{at center} \Rightarrow 2m \quad g \]

No rotation \( \Rightarrow \sum T = 0 \)

\( \vec{w} \) tries to rotate clockwise while \( \vec{n} \) counterclockwise

\( \Rightarrow \sum T = T_n - T_w = 0 \quad \Rightarrow T_n = T_w \)

\( \vec{n} \) horizontal, \( \vec{w} \) vertical \( \Rightarrow \gamma n = xw \)

\( \Rightarrow (3m)(75N) = (2m)w \quad \Rightarrow w = \frac{(75N)(3)}{2} = 112.5N \)
10. A man, sitting on the office chair shown, has two large weights in his hands. The man is spinning at $3 \text{ rad/s}$ with the weights held out to the sides. Which of the following is a possible angular speed for the man after he has pulled the weights closely to his chest?

\[ \text{Conservation of Angular Momentum with Single Object} \]

\[ I_1 \omega_1 = I_2 \omega_2 \]

$\omega_1 = 3 \text{ rad/s}$. Bringing weights in $\Rightarrow$

$I_2 < I_1$ so $\omega_2 > \omega_1$
(11.) A 3 kg mass is attached to a 250 N/m spring as shown below. At time \( t = 0 \), the mass is started from rest some distance from its equilibrium position. Where should the mass be started to give its motion a period of 1.5 \( s \)? Assume there is no friction between the mass and the floor.

\[
\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250 \text{ N/m}}{3 \text{ kg}}} = 9.128 \text{ rad/s}
\]

\[
\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{9.128 \text{ rad/s}} = 0.688 \text{ s}
\]

Period already determined and no choice of initial position can change it.

\[
\begin{array}{|c|}
\hline
\text{(a) } 0.43 \text{ m} \\
\text{(b) } 0.69 \text{ m} \\
\text{(c) } 2.3 \text{ m} \\
\text{(d) Any distance will give a period of 1.5 s.} \\
\text{(e) There is no distance that will give a period of 1.5 s.} \\
\hline
\end{array}
\]
(12.) A 2 kg mass is attached to a 50 N/m spring as shown below. At time \( t = 0 \), the mass is started from its equilibrium position with a velocity of 5.6 m/s to the right. There is no friction between the mass and the floor. What is the phase angle, \( \phi \), in the equation \( x = A \cos (\omega t + \phi) \) for this motion? (Answer: to the right positive)

\[
x = A \cos (\omega t + \phi)
\]

\[\Rightarrow x_0 = A \cos \phi\]

\[x_0 = 0 \Rightarrow 0 = A \cos \phi \Rightarrow \phi = \pm \frac{\pi}{2}\]

\[v = -\omega A \sin (\omega t + \phi) \Rightarrow v_0 = -\omega A \sin \phi\]

\(v_0 \) positive \( \Rightarrow \phi \) must be \( \pm \frac{\pi}{2} \) such that

\[v_0 = -\omega A \sin (-\frac{\pi}{2}) = -\omega A (1) = -\omega A\]
(13.) On some alien planet, you find that a 0.34-m long simple pendulum has a period of 1.1 s, what is the acceleration due to gravity on that planet?

\[
\text{Simple Pendulum } \Rightarrow \omega = \sqrt{\frac{g}{L}}
\]

\[
T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi \sqrt{\frac{L}{g}}
\]

\[
\therefore T^2 = \frac{4\pi^2 L}{g} \Rightarrow g = \frac{4\pi^2 L}{T^2}
\]

\[
\therefore g = \frac{4\pi^2 (0.34\text{ m})}{(1.1\text{ s})^2} = 11.08 \text{ m/s}^2 = 11\text{ m/s}^2
\]
(14.) Which pair of graphs shown below correspond to a sinusoidal wave with a speed of \( v = 1.6 \text{ cm/s} \)?

(a) Graphs #2 and #1.
(b) Graphs #3 and #1.
(c) Graphs #4 and #1.
(d) Graphs #3 and #2.
(e) Graphs #4 and #2.

\[ V = \frac{x}{t} \]

\[ V_{31} = \frac{60 \text{ cm}}{40 \text{ s}} = 1.5 \text{ cm/s} \]

\[ V_{41} = \frac{32 \text{ cm}}{40 \text{ s}} = 0.8 \text{ cm/s} \]

\[ V_{32} = \frac{60 \text{ cm}}{50 \text{ s}} = 1.28 \text{ cm/s} \]

\[ V_{42} = \frac{32 \text{ cm}}{50 \text{ s}} = 0.64 \text{ cm/s} \]
(15.) Your starship, *The Aimless Wanderer*, is in circular orbit around a \(5.0 \times 10^6\text{-m}\)-radius, alien planet (which by law you have to call Mongo) with a period of 5.00 hours. If *The Aimless Wanderer’s* distance from the center of Mongo is \(1.1 \times 10^7\text{ m}\), what is the acceleration due to gravity on the surface of planet Mongo?

\[
g = \frac{GM_p}{R_p^2}
\]

\[R_p = 5 \times 10^6\text{ m but need } M_p\]

\[T = \frac{2\pi R_p^3}{GM_p}\]

\[T = 5\text{ h} \times \frac{3600\text{ s}}{\text{h}} = 18000\text{ s}\]

\[\frac{T^2}{4\pi^2} = \frac{\frac{4\pi^2}{G M_p}}{R_p^3} = \frac{4\pi^2 (1.1 \times 10^7\text{ m})^3}{(0.07 \times 1.0 \times 2/3)(18000)^3} = 2.43 \times 10^9\text{ kg}\]

\[g = \frac{GM_p}{R_p^2} = (0.07 \times 10^{-11}\text{ N m}^2/\text{kg}^2)(2.43 \times 10^9\text{ kg})
\[
\frac{(5 \times 10^6\text{ m})^2}{(5 \times 10^6\text{ m})^2} = 0.487\text{ m/s}^2
\]

\[= 0.5\text{ m/s}^2\]